Control Theory and Viability Methods for the Sustainable Management of Natural Resources

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We highlight management issues at the interface between nature and society

To make a long story short . . .

We claim that mathematical control theory is an insightful framework to deal with natural resources management issues

Problems. Many natural resources management problems can be grasped within mathematical control theory

- climate change mitigation, management of energies, etc.
- fisheries management, epidemics control, etc.

Methods. Theory provides concepts, tools and methods

- viability kernel, viable controls
- dynamic programming, monotonicity

Answers. Practical answers are obtained

- ecosystem viable yields, precautionary rules
- tradeoffs display between economic and ecological sustainability thresholds and risk

I travel with colleagues along this journey

- C. Béné, L. Doyen, and D. Gabay. *A viability analysis for a bio-economic model.* Ecological Economics, 36:385–396, 2001.
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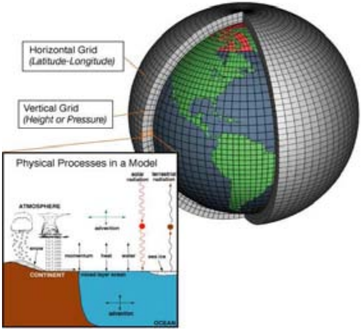
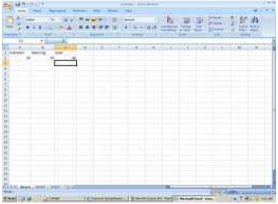
- M. De Lara, L. Doyen, T. Guilbaud, M.-J. Rochet. *Monotonicity properties for the viable control of discrete time systems* Systems and Control Letters Volume 56, Number 4, 2006, Pages 296-302.
- V. Martinet and L. Doyen. *Sustainable management of an exhaustible resource: a viable control approach.* Resource and Energy Economics, 29(1):p.17–39, 2007.
- V. Martinet, L. Doyen, and O. Thébaud. *Defining viable recovery paths toward sustainable fisheries.* Ecological Economics, 64(2):411–422, 2007.
- L. Doyen, M. De Lara, J. Ferraris, and D. Pelletier.*Sustainability of exploited marine ecosystems through protected areas: a viability model and a coral reef case study.* Ecological Modelling, 208(2-4):353–366, November 2007.
- M. De Lara and V. Martinet.*Multi-criteria dynamic decision under uncertainty: a stochastic viability analysis and an application to sustainable fishery management*. Mathematical Biosciences, Volume 217, Issue 2, February 2009, Pages 118-124.
- M. De Lara, P. Gajardo, H. Ramírez C. *Viable harvest of monotone bioeconomic models.* Systems and Control Letters, 2010 Volume 60, Pages 192-197, 2011.
- L. Doyen, M. De Lara.*Stochastic viability and dynamic programming*. Systems and Control Letters, Volume 59, Number 10, Pages 629-634, 2010.
- Michel De Lara, Eladio Ocana Anaya, Ricardo Oliveros–Ramos, Jorge Tam. *Ecosystem Viable Yields.* Environmental Modeling & Assessment, 2012.

Outline of the presentation

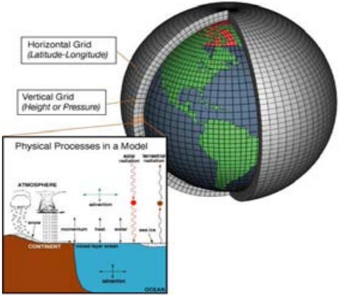
1. Natural resources management issues and viability
   - Examples of decision models
   - Discrete–time viability
   - Are the ICES fishing quotas recommendations "sustainable"?
   - Ecosystem viable yields (anchovy–hake application)
2. Risk management, robust and stochastic viability
   - Uncertain systems, policies and viable scenarios
   - Robust viability
   - Robust viability kernel (anchovy–hake application)
   - Stochastic viability
   - Bycatches in a nephrops-hake fishery
   - Dam management under environmental/tourism constraint
3. Contribution to quantitative sustainable management

Outline of the presentation





We distinguish two polar classes of models: knowledge models *versus* decision models



Knowledge models:
$1/1\ 000\ 000 \rightarrow 1/1\ 000 \rightarrow 1/1$ maps

Office of Oceanic and Atmospheric Research (OAR) climate model

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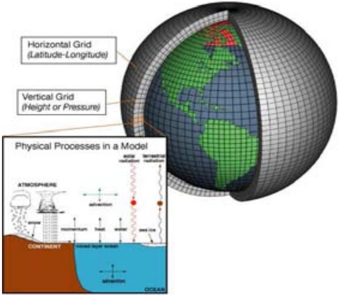

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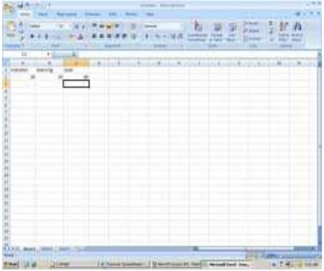


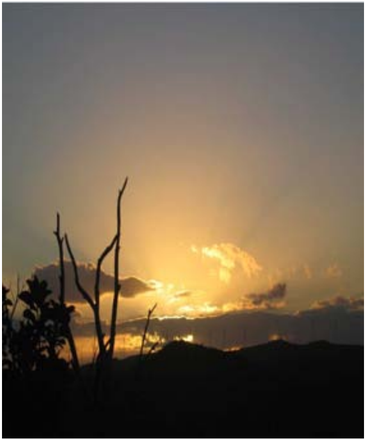
Action/decision models:
economic models are fables designed to provide insight

William Nordhaus economic-climate model

This talk is *not* about crafting dynamical models

- Elaborating a dynamical model is a delicate venture
  - Peter Yodzis
  - Carlos Castillo Chavez
- Our starting point will be a mathematical dynamical model that captures how sequences of decisions affect a "piece of reality"
- Then, we will use such a model to frame a decision problem

Let us scout a very stylized model of the climate-economy system



We lay out a dynamical model with

- two state variables

  environmental: atmospheric CO_2 concentration level $M(t)$

  economic: gross world product GWP $Q(t)$

- one decision variable, the emission abatement rate $a(t)$

A carbon cycle model "à la Nordhaus" is an example of *decision model*

- Time index t in years
- Economic production $Q(t)$ (GWP)

$$Q(t+1) = \overbrace{(1+g)}^{\text{economic growth}} Q(t)$$

- CO_2 concentration $M(t)$

$$M(t+1) = M(t) \underbrace{-\delta(M(t) - M_{-\infty})}_{\text{natural sinks}} + \alpha \overbrace{\mathtt{Emiss}(Q(t))}^{\text{emissions}} \underbrace{(1 - a(t))}_{\text{abatement}}$$

- Decision $a(t) \in [0, 1]$ is the abatement rate of CO_2 emissions

Data

- $M(t)$ CO_2 atmospheric concentration, measured in ppm, parts per million (379 ppm in 2005)
- $M_{-\infty}$ pre-industrial atmospheric concentration (about 280 ppm)
- $\texttt{Emiss}(Q(t))$ "business as usual" CO_2 emissions (about 7.2 GtC per year between 2000 and 2005)
- $0 \leq a(t) \leq 1$ abatement rate reduction of CO_2 emissions
- α conversion factor from emissions to concentration ($\alpha \approx 0.471$ ppm.GtC^{-1} sums up highly complex physical mechanisms)
- δ natural rate of removal of atmospheric CO_2 to unspecified sinks ($\delta \approx 0.01$ year^{-1})

A concentration target is pursued to avoid danger

Limitation of concentrations of $\mathrm{CO_2}$

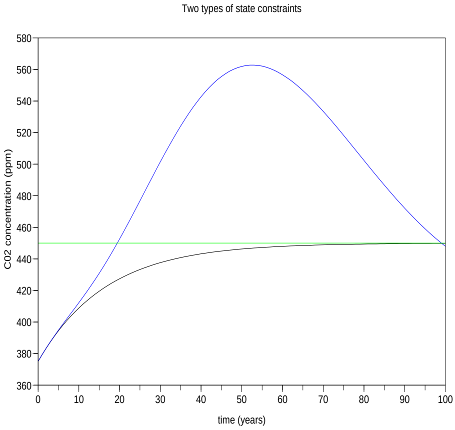
- below a tolerable threshold $M^\sharp$ (say 350 ppm, 450 ppm)
- at a specified date $T > 0$ (say year 2050 or 2100)

United Nations Framework Convention on Climate Change

"to achieve, (...), stabilization of greenhouse gas concentrations in the atmosphere at a level that would prevent dangerous anthropogenic interference with the climate system"

$$\underbrace{M(T)}_{\text{concentration at horizon}} \leq \underbrace{M^\sharp}_{\text{threshold}}$$

Constraints capture different requirements



- The concentration has to remain below a tolerable level at the horizon T:

$$M(T) \leq M^{\sharp}$$

- More demanding: from the initial time t_0 up to the horizon T

$$M(t) \leq M^{\sharp}$$

$$t = t_0, \ldots, T$$

Constraints may be environmental, physical, economic

- The concentration has to remain below a tolerable level from initial time t_0 up to the horizon T

$$M(t) \leq M^{\sharp}, \quad t = t_0, \ldots, T$$

- Abatements are expressed as fractions

$$0 \leq a(t) \leq 1, \quad t = t_0, \ldots, T-1$$

- As with "cap and trade", setting a ceiling on CO_2 price amounts to cap abatement costs

$$\underbrace{\texttt{Cost}\big(a(t), Q(t)\big)}_{\text{costs}} \leq c^{\sharp} \; (100 \text{ euros / tonne } CO_2), \quad t = t_0, \ldots, T-1$$

Mixing dynamics, optimization and constraints yields a cost-effectiveness problem

- Minimize abatement costs

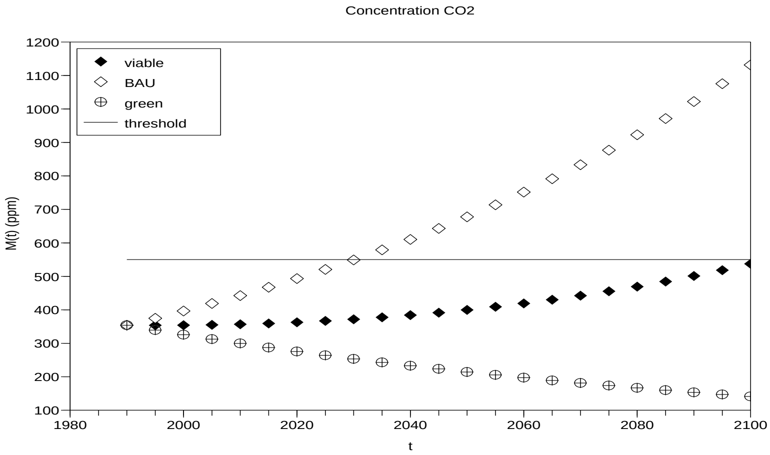
$$\min_{a(t_0),\dots,a(T-1)} \sum_{t=t_0}^{T-1} \Big(\frac{1}{1+r_e}\Big)^{t-t_0} \underbrace{\texttt{Cost}\big(a(t), Q(t)\big)}_{\text{abatement costs}}$$

- under the GWP-CO_2 dynamics

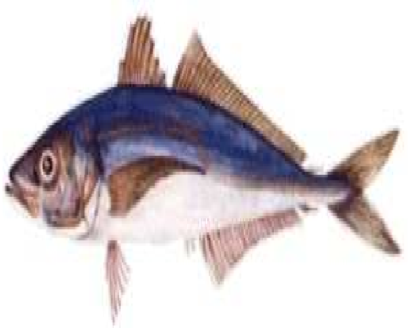
$$\begin{cases} M(t+1) & = M(t) - \delta(M(t) - M_{-\infty}) + \alpha \texttt{Emiss}\big(Q(t)\big)(1-a(t)) \\ Q(t+1) & = (1+g)Q(t) \end{cases}$$

- and under target constraint

$$\underbrace{M(T) \leq M^{\sharp}}_{\text{CO2 concentration}}$$


Concentration CO2
viable
BAU
green
threshold
M(t) (ppm)
1200
1100
1000
900
800
700
600
500
400
300
200
100
1980
2000
2020
2040
2060
2080
2100
t

Populations can be described by abundances at ages



Jack Mackrel abundances (Chilean data)
are measured in thousand of individuals

13651022	thousand of age < 1 (recruits)
7495888	thousand of age $\in [1, 2[$
6804151	
4191318	
4582943	
2500338	
1139182	
523261	
269328	
166390	
95606	thousand of age ≥ 11

We now line up the ingredients of a harvested population age-class dynamical model

- Time $t \in \mathbb{N}$ measured in years
- Abundances at age $N = (N_a)_{a=1,\ldots,A} \in \mathbb{X} = \mathbb{R}_+^A$
- $a \in \{1, \ldots, A\}$ age class index
  - $A = 3$ for anchovy
  - $A = 8$ for hake
  - $A = 40$ for bacalao
- Control variable $\lambda \in \mathbb{U} = \mathbb{R}_+$ is fishing effort

One year older every year...

Except for the recruits ($a = 1$) and the last age class ($a = A$),

$$N_a(t+1) = e^{-\overbrace{(\underbrace{M_{a-1}}_{\text{natural}} + \underbrace{\lambda(t)F_{a-1}}_{\text{fishing}})}^{\text{mortality}}} N_{a-1}(t), \quad a = 2, \dots, A-1$$

where

- M_a stands for the natural mortality-at-age a
- F_a is the harvesting mortality rate of individuals of age a, also called exploitation pattern-at-age a, related to the mesh size for instance
- the control variable $\lambda(t)$ is the fishing effort, or the exploitation pattern multiplier

The last age-class may comprise a plus-group

N_A is the abundance of individuals of age above $A-1$.

$$\begin{aligned} N_A(t+1) &= N_{A-1}(t)\exp\big(-(M_{A-1}+\lambda(t)F_{A-1})\big) \\ &+ \underbrace{\pi}_{0 \text{ or } 1} N_A(t)\exp\big(-(M_A+\lambda(t)F_A)\big) \end{aligned}$$

The parameter $\pi \in \{0,1\}$ is related to the existence of a so-called plus-group:

- if we neglect the survivors older than age A then $\pi = 0$ (for anchovy)
- else $\pi = 1$, and the last age class is a plus group (for hake)

The stock-recruitment function mathematically turns spawning stock biomass into future recruits abundance

- The spawning stock biomass is

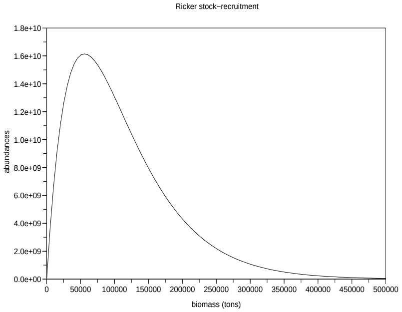
$$\mathrm{SSB}(N) = \sum_{a=1}^{A} \underbrace{\gamma_a}_{\text{proportion}} \overbrace{\mu_a}^{mass} \underbrace{N_a}_{\text{abundance}}$$

  with
  - γ_a proportion of matures-at-age a
  - μ_a weight-at-age a
- stock-recruitment relationship $\mathtt{S/R}$: biomass $\to$ abundance

$$\underbrace{N_1(t+1)}_{\text{future recruits}} = \mathtt{S/R}\Big(\underbrace{\mathtt{SSB}\big(N(t)\big)}_{\text{spawning biomass}} \Big)$$

Here are traditional examples of stock-recruitment functions

Recruitment involves complex biological and environmental processes that fluctuate in time, and are difficult to integrate into a population model



- constant: $\mathtt{S/R}(B) = R$
- linear: $\mathtt{S/R}(B) = rB$
- Beverton-Holt: $\mathtt{S/R}(B) = \frac{B}{\alpha+\beta B}$
- Ricker: $\mathtt{S/R}(B) = \alpha B \mathrm{e}^{-\beta B}$

And here are the state vector and the control

- The state vector $N(t)$ is forged with abundances at age

$$N(t) = \begin{pmatrix} N_1(t) \\ N_2(t) \\ \vdots \\ N_{A-1}(t) \\ N_A(t) \end{pmatrix} \in \mathbb{R}_+^A$$

- The scalar control $\lambda(t)$ is the fishing effort multiplier

A harvested population age-class model is an A—dimensional controlled dynamical system

$$N_1(t+1) = \mathtt{S/R}\Big(\overbrace{\mathtt{SSB}(N(t))}^{\text{spawning biomass}}\Big) \quad \text{recruitment}$$

$$N_2(t+1) = e^{-(M_1+\lambda(t)F_1)} N_1(t)$$

$$N_a(t+1) = e^{-\overbrace{(\underbrace{M_{a-1}}_{\text{natural}} + \underbrace{\lambda(t)F_{a-1}}_{\text{fishing}})}^{\text{mortality}}} N_{a-1}(t), \quad a = 2, \dots, A-1$$

$$N_{A-1}(t+1) = e^{-(M_{A-2}+\lambda(t)F_{A-2})} N_{A-2}(t)$$

$$N_A(t+1) = e^{-(M_{A-1}+\lambda(t)F_{A-1})} N_{A-1}(t) + \underbrace{\pi e^{-(M_A+\lambda(t)F_A)}}_{\text{plus group}} N_A(t)$$

The ICES precautionary approach uses indicators and reference points to tackle ecological objectives

International Council for the Exploration of the Sea precautionary approach

- keeping (or restoring) spawning stock biomass SSB indicator above a threshold reference point B_{lim}
- restricting fishing effort to have mean fishing mortality F indicator below a threshold reference point F_{lim}

Definition	**Notation**	**Anchovy**	**Hake**
F limit RP	F_{lim}	/	0.35
SSB limit RP (t)	B_{lim}	21 000	100 000

Spawning biomass and fishing mortality are outputs of the harvested population age-class model

- Spawning stock biomass

$$\mathrm{SSB}(N) = \sum_{a=1}^{A} \underbrace{\gamma_a}_{\text{proportion}} \overbrace{\mu_a}^{\text{mass}} \underbrace{N_a}_{\text{abundance}}$$

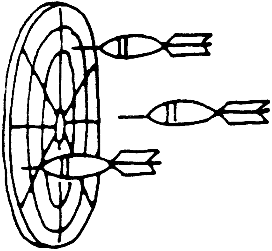
with reference point $\mathrm{SSB}(N) \geq B_{\text{lim}}$

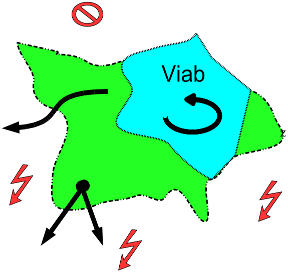
- Mean fishing mortality over age range from a_r to A_r

$$F(\lambda) := \frac{\lambda}{A_r - a_r + 1} \sum_{a=a_r}^{a=A_r} F_a$$

with reference point $F(\lambda) \leq F_{\text{lim}}$

Outline of the presentation





A control system connects input and output variables

Input variables

Control	wood logs
Uncertainty	wood humidity
	metal conductivity

Output variables

soup quality
water vapor
temperature (internal state)

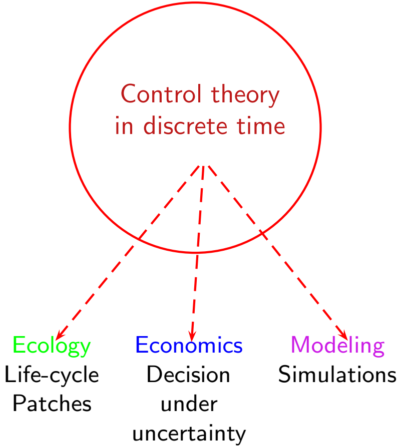
Discrete-time nonlinear state-control systems are special input-output systems

A specific output is distinguished, and is labeled state,
when the system may be written as

$$x(t+1) = \mathtt{Dyn}(t, x(t), u(t)), \quad t \in \mathbb{T} = \{t_0, t_0+1, \ldots, T-1\}$$

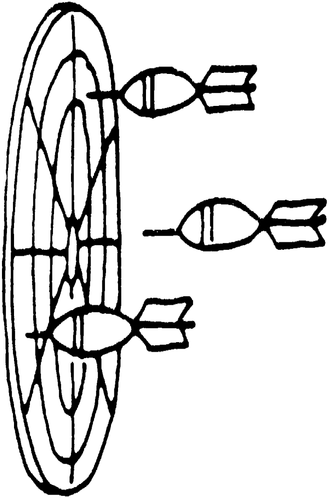
- the time $t \in \overline{\mathbb{T}} = \{t_0, t_0+1, \ldots, T-1, T\} \subset \mathbb{N}$ is discrete with initial time t_0 and horizon T ($T < +\infty$ or $T = +\infty$) (*the time period* $[t, t+1[$ *may be a year, a month, etc.*)
- the state variable $x(t)$ belongs to the finite dimensional *state space* $\mathbb{X} = \mathbb{R}^{n_{\mathbb{X}}}$; (*biomasses, abundances, capital, etc.*)
- the control variable $u(t)$ is an element of the *control space* $\mathbb{U} = \mathbb{R}^{n_{\mathbb{U}}}$ (*catches, harvesting effort, investment, etc.*)
- the dynamics $\mathtt{Dyn}$ maps $\mathbb{T} \times \mathbb{X} \times \mathbb{U}$ into $\mathbb{X}$ (*age-class model, population dynamics, economic model, etc.*)

We dress natural resources management issues in the formal clothes of control theory in discrete time



- Problems are framed as
  - find controls/decisions driving a dynamical system
  - to achieve various goals
- Three main ingredients are
  - controlled dynamics
  - constraints
  - criterion to optimize

We mathematically express the objectives pursued as control and state constraints



- For a state-control system, we cloth objectives as constraints
- and we distinguish
  control constraints (rather easy)
  state constraints (rather difficult)
- Viability theory deals with state constraints

Constraints may be explicit on the control variable

and are rather easily handled by reducing the decision set

Examples of control constraints

- Irreversibility constraints, physical bounds
  $0 \leq a(t) \leq 1$, $0 \leq h(t) \leq B(t)$, $0 \leq q(t) \leq \min\{S(t), q^\sharp\}$,
- Tolerable costs $c\big(a(t), Q(t)\big) \leq c^\sharp$

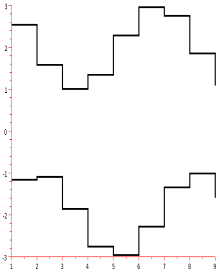
Control constraints / admissible decisions

$$\underbrace{u(t)}_{\text{control}} \in \underbrace{\mathbb{B}\big(t, x(t)\big)}_{\text{admissible set}} , \quad t = t_0, \ldots, T-1$$

Easy because control variables $u(t)$ are precisely those variables whose values the decision-maker can fix at any time within given bounds

Meeting constraints bearing on the state variable is delicate

due to the dynamics pipeline between controls and state



State constraints / admissible states

$$\underbrace{x(t)}_{\text{state}} \in \underbrace{\mathbb{A}(t)}_{\text{admissible set}} \quad , \quad t = t_0, \ldots, T$$

Examples ("tipping points")

- CO_2 concentration $M(t) \leq M^\sharp$
- biomass $B^\flat \leq B(t) \leq B^\sharp$

State constraints are mathematically difficult because of "inertia"

$$x(t) = \underbrace{\text{function}}_{\text{iterated dynamics}} \big(\underbrace{u(t-1), \ldots, u(t_0)}_{\text{past controls}}, x(t_0) \big)$$

Target and asymptotic state constraints are special cases

- Final state achieves some target

$$\underbrace{x(T)}_{\text{final state}} \in \underbrace{\mathbb{A}(T)}_{\text{target set}}$$

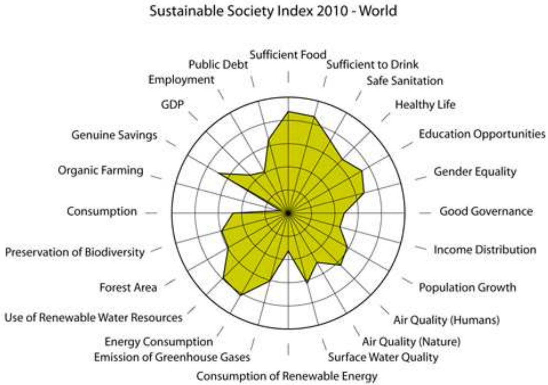
Example: CO_2 concentration

- State converges toward a target

$$\underbrace{\lim_{t\to+\infty} x(t)}_{\text{asymptotic state}} \in \underbrace{\mathbb{A}(\infty)}_{\text{target set}}$$

Example: convergence towards an endemic state in epidemiology

Some economists recommend objectives to be expressed in their own units, without aggregation

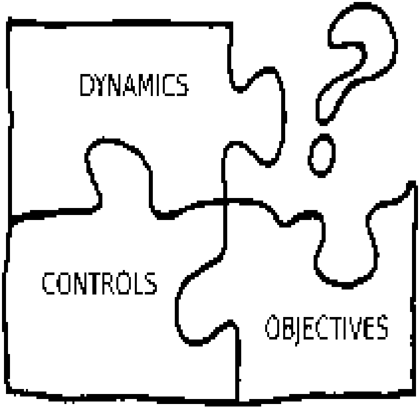


The “Stiglitz-Sen-Fitoussi” Commission (2009) déconseille de privilégier un indicateur synthétique unique car, quel que soit l’indicateur envisagé, l’agrégation de données disparates ne va pas de soi

When dealing with economic and environmental objectives, this disaggregated approach is coined co-viability

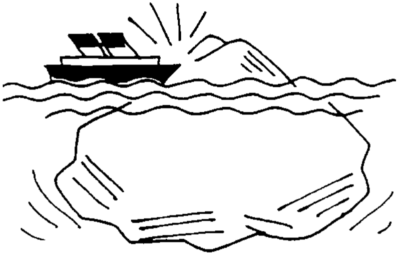
- Co-viability when
  - environmental constraints: conservation, viability
  - economic constraints: production, efficiency
- C. Béné, L. Doyen, and D. Gabay
  A viability analysis for a bio-economic model.
  Ecological Economics, 36:385–396, 2001

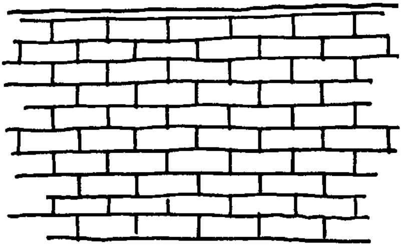
Can we solve the compatibility puzzle between dynamics and objectives by means of appropriate controls?



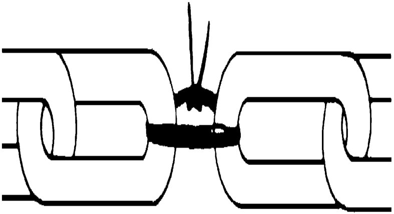
- Given a dynamics that mathematically embodies the causal impact of controls on the state
- Imposing objectives bearing on output variables (states, controls)
- Is it possible to find a control path that achieves the objectives for all times?

Crisis occurs when constraints are trespassed at least once





- An initial state is not viable if, whatever the sequence of controls, a crisis occurs
- There exists a time when one of the state or control constraints is violated

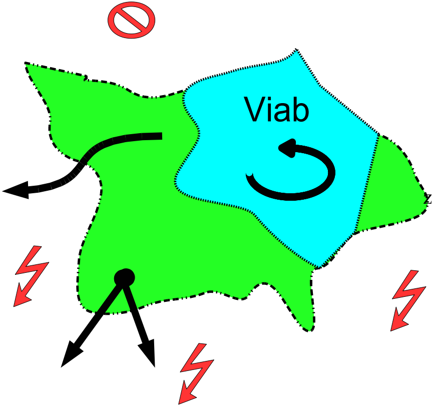


The compatibility puzzle can be solved when the initial viability kernel $\mathbb{V}\mathrm{iab}(t_0)$ is not empty

Viable initial states form the viability kernel (Jean-Pierre Aubin)

$$
\mathbb{V}\mathrm{iab}(t) := \left\{ \begin{array}{c|l} & \text{there exist a control path } u(\cdot) = \\ & \big(u(t), u(t+1), \ldots, u(T-1)\big) \\ & \text{and a state path } x(\cdot) = \\ & \big(x(t), x(t+1), \ldots, x(T)\big) \\ \text{initial} & \text{starting from } x(t) = x \text{ at time } t \\ \text{states} & \text{satisfying for any time } s \in \{t, \ldots, T-1\} \\ x \in \mathbb{X} & x(s+1) = \mathtt{Dyn}\big(s, x(s), u(s)\big) \quad \textit{dynamics} \\ & u(s) \in \mathbb{B}(s, x(s)) \quad \textit{control constraints} \\ & x(s) \in \mathbb{A}(s) \quad \textit{state constraints} \\ & \text{and } x(T) \in \mathbb{A}(T) \quad \textit{target constraints} \end{array} \right\}
$$

The viability kernel is included in the state constraint set



- The largest set is the state constraint set $\mathbb{A}$
- It includes the smaller blue viability kernel $\mathbb{V}\mathrm{iab}(t_0)$
- The green set measures the incompatibility between dynamics and constraints: good start, but inevitable crisis!

The viability program aims at turning a priori constraints, with state constraints, into a posteriori constraints, without state constraints

- A priori constraints, with state constraints

$$\begin{cases} x(t_0) \in \mathbb{X} \\ x(t+1) = \mathtt{Dyn}\big(t, x(t), u(t)\big) \\ u(t) \in \mathbb{B}\big(t, x(t)\big) & \text{control constraints} \\ x(t) \in \mathbb{A}(t) & \text{state constraints} \end{cases}$$

- are turned into a posteriori constraints, without state constraints except for the initial state

$$\begin{cases} x(t_0) \in \mathbb{V}\mathrm{iab}(t_0) & \text{initial state constraint} \\ x(t+1) = \mathtt{Dyn}\big(t, x(t), u(t)\big) \\ u(t) \in \mathbb{B}^{\mathsf{viab}}\big(t, x(t)\big) & \text{control constraints} \end{cases}$$

The viability kernels satisfy a backward dynamic programming equation

Proposition

Assume that $T < +\infty$. The viability kernels $\mathbb{V}\mathrm{iab}(t)$ satisfy a backward induction, where t runs from $T-1$ down to t_0:

$$\mathbb{V}\mathrm{iab}(T) = \mathbb{A}(T)$$

$$\begin{aligned}\mathbb{V}\mathrm{iab}(t) = \{ & \textit{admissible states } x \in \mathbb{A}(t) \mid \\ & \textit{there exists an admissible control } u \in \mathbb{B}(t,x) \\ & \textit{such that the future state } \mathtt{Dyn}(t,x,u) \\ & \textit{belongs to the next viability kernel } \mathbb{V}\mathrm{iab}(t+1) \ \}\end{aligned}$$

The dynamic programming equation yields viable controls

- The following viable regulation set

$$\mathbb{B}^{\text{viab}}(t,x) := \{u \in \mathbb{B}(t,x) \mid \mathtt{Dyn}(t,x,u) \in \mathbb{V}\mathrm{iab}(t+1)\}$$

is not empty if and only if $x \in \mathbb{V}\mathrm{iab}(t)$

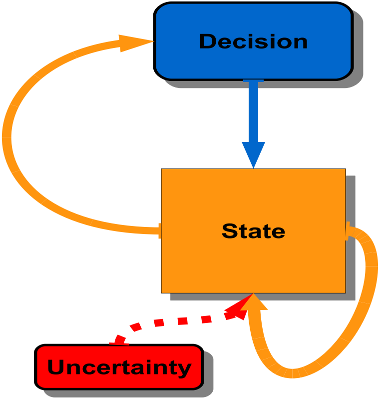
$$\mathbb{B}^{\text{viab}}(t,x) \neq \emptyset \iff x \in \mathbb{V}\mathrm{iab}(t)$$

- Any $u \in \mathbb{B}^{\text{viab}}(t,x)$ is said to be a viable control
- A viable policy is a mapping $\mathtt{Pol} : \mathbb{T} \times \mathbb{X} \to \mathbb{U}$ such that

$$\mathtt{Pol}(t,x) \in \mathbb{B}^{\text{viab}}(t,x)$$

for all $(t,x) \in \mathbb{T} \times \mathbb{X}$

"Policies" are closed-loop controls



- Deterministic control theory appeals to open-loop control, ⌚ that is, a time-dependent sequence (planning, scheduling)

$$u : \underbrace{t \in \mathbb{T}}_{\text{time}} \mapsto \underbrace{u(t) \in \mathbb{U}}_{\text{control}}$$

- Another notion of solution is a decision rule, ⌚× 👁 a policy, that is, a mapping

$$\mathtt{Pol} : \underbrace{(t, x) \in \mathbb{T} \times \mathbb{X}}_{\text{(time, state)}} \mapsto u = \underbrace{\mathtt{Pol}(t, x) \in \mathbb{U}}_{\text{control}}$$

which "closes the loop" between time t–state x and control u (and is especially relevant in presence of uncertainties)

Monotonicity assumptions on dynamics and constraints can help identify viable decision rules

Monotonicity assumptions

- Dynamics Dyn is monotonous:
  - the more abundant today, the more tomorrow
  - the more harvested today, the less abundance tomorrow (monospecific models and technical interactions)
- Constraints/objectives are monotonous functions

Results

- Lower and upper approximations of the viability kernel
- Precautionary viable decision rules

Outline of the presentation

Is the ICES precautionary approach sustainable?

- The precautionary approach (PA) may be sketched as follows
  - the condition $\texttt{SSB}(N) \geq B_{\lim}$ is checked
  - if valid, the following usual advice is given

$$\underbrace{\lambda_{UA} \overbrace{(N)}^{\text{abundance}}}_{\text{effort}} = \max\{\lambda \in \mathbb{R}_+ \mid \overbrace{\texttt{SSB}(\texttt{Dyn}(N,\lambda))}^{\text{next year spawning biomass}} \geq B_{\lim} \text{ and } \underbrace{F(\lambda)}_{\text{fishing mortality}} \leq F_{\lim}\}$$

- Is it possible to apply the ICES precautionary rule every year?
- If so, can we remain within precautionary bounds as follows?

$$\texttt{SSB}(N(t)) \geq B_{\lim} \text{ and } F(\lambda(t)) \leq F_{\lim} , \quad \forall t = t_0, t_0+1, \ldots$$

The ICES precautionary rule is sustainable or not, depending on the stock-recruitment model

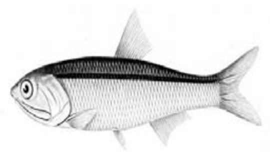
- Bay of Biscay anchovy

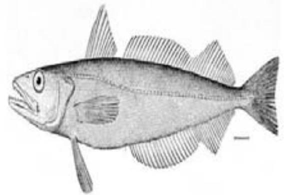
S/R Relationship	Constant	Constant	Constant (2002)	Constant (2004)	Linear	Ricker
Condition	$R_{mean} \geq \underline{R}$	$R_{gm} \geq \underline{R}$	$R_{min} \geq \underline{R}$	$R_{min} \geq \underline{R}$	$\gamma_1 \mu_1 r \geq 1$	
Left hand side	$14\ 016\ \times 10^6$	$7\ 109\ \times 10^6$	$3\ 964\ \times 10^6$	$696\ \times 10^6$	0.84	0
Right hand side	$1\ 312\ \times 10^6$	$1\ 312\ \times 10^6$	$1\ 312\ \times 10^6$	$1\ 312\ \times 10^6$	1	21 000
Sustainable	yes	yes	yes	no	no	no

- For species with late maturation, like hake, ICES precautionary approach is never sustainable!

Outline of the presentation

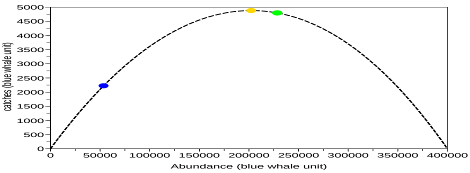
1. Natural resources management issues and viability
   - Examples of decision models
   - Discrete–time viability
   - Are the ICES fishing quotas recommendations "sustainable"?
   - Ecosystem viable yields (anchovy–hake application)
2. Risk management, robust and stochastic viability
   - Uncertain systems, policies and viable scenarios
   - Robust viability
   - Robust viability kernel (anchovy–hake application)
   - Stochastic viability
   - Bycatches in a nephrops-hake fishery
   - Dam management under environmental/tourism constraint
3. Contribution to quantitative sustainable management



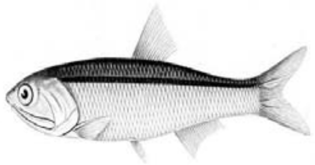


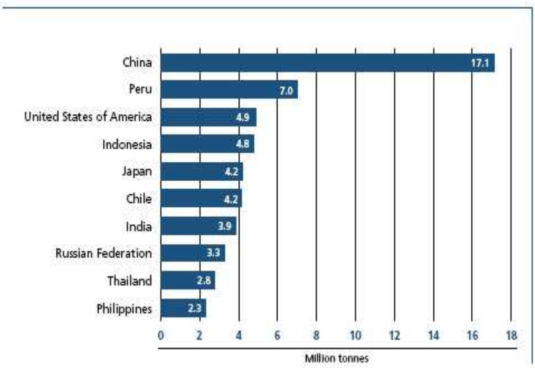
Despite calls to an "ecosystem approach", stocks management remains monospecific

- The World Summit on Sustainable Development (Johannesburg, 2002) encouraged the application of the "ecosystem approach" by 2010
- but... following the Summit, the signatory States undertook to restore and exploit their stocks at maximum sustainable yield (MSY)
- The MSY is a concept which relies upon a monospecific dynamic model $\dot{B} = f(B) - qEB$ where B is biomass, and E fishing effort



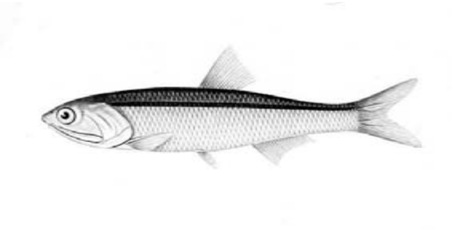
Perú is World 2nd for marine and inland capture fisheries

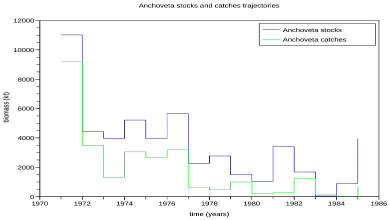


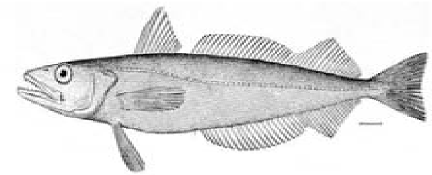


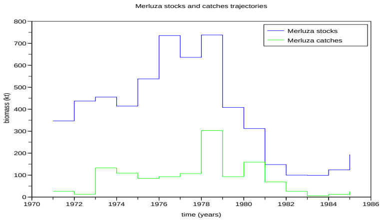
The northern Humboldt current system off Perú covers less than 0.1% of the world ocean
but presently sustains
about 10% of the world fish catch

We were lucky enough that IMARPE entrusted us yearly data of anchoveta and merluza stock and catches from 1971 to 1985









We consider two species targeted by two fleets in a biomass ecosystem dynamic

We embody stocks and fishing interactions
in a two-dimensional dynamical model

$$\overbrace{A(t+1)}^{\text{future biomass}} = A(t)\overbrace{\mathcal{R}_A\big(A(t),H(t)\big)}^{\text{growth factor}}\big(1-\underbrace{E_A(t)}_{\text{effort}}\big)$$

$$H(t+1) = H(t)\mathcal{R}_H\big(A(t),H(t)\big)\big(1-\overbrace{E_H(t)}^{\text{control}}\big)$$

- State vector $(A(t),H(t))$ represents biomasses
- Control vector $(E_A(t),E_H(t))$ is fishing effort of each species
- Catches are $E_A(t)\mathcal{R}_A\big(A(t),H(t)\big)A(t)$ and $E_H(t)\mathcal{R}_H\big(A(t),H(t)\big)H(t)$ (measured in biomass)

Our objectives are twofold: conservation and production

The viability kernel is the set of initial species biomasses $\big(A(t_0), H(t_0)\big)$ from which appropriate effort controls $\big(E_A(t), E_H(t)\big)$, $t = t_0, t_0+1, \ldots$ produce a trajectory of biomasses $\big(A(t), H(t)\big)$, $t = t_0, t_0+1, \ldots$ such that the following goals are satisfied

- preservation (minimal biomass thresholds)

$$A \text{ stocks}: \qquad A(t) \geq S_A^\flat$$

$$H \text{ stocks}: \qquad H(t) \geq S_H^\flat$$

- economic/social requirements (minimal catch thresholds)

$$A \text{ catches}: \qquad E_A(t)\mathcal{R}_A\big(A(t), H(t)\big)A(t) \geq C_A^\flat$$

$$H \text{ catches}: \qquad E_H(t)\mathcal{R}_H\big(A(t), H(t)\big)H(t) \geq C_H^\flat$$

We provide an explicit expression for the viability kernel under rather weak assumptions

Proposition

If the thresholds $S_A^\flat, S_H^\flat$ and $C_A^\flat, C_H^\flat$ meet the inequalities

$$\underbrace{S_A^\flat \mathcal{R}_A(S_A^\flat, S_H^\flat) - S_A^\flat}_{\text{surplus}} \geq C_A^\flat \quad \text{and} \quad \underbrace{S_H^\flat \mathcal{R}_H(S_A^\flat, S_H^\flat) - S_H^\flat}_{\text{surplus}} \geq C_H^\flat$$

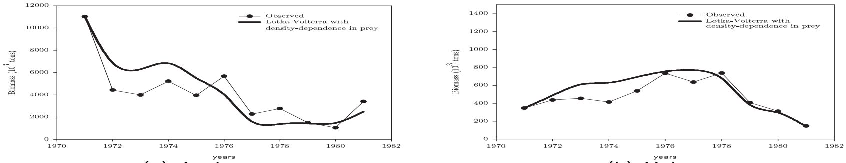
the viability kernel is given by

$$\Big\{(A, H) \mid A \geq S_A^\flat,\ H \geq S_A^\flat,\ A\mathcal{R}_A(A, H) - S_A^\flat \geq C_A^\flat,\ H\mathcal{R}_H(A, H) - S_H^\flat \geq C_H^\flat\Big\}$$

We taylor a Lotka-Volterra *decision model* to hake-anchovy Peruvian fisheries scarce data

Hake-anchovy Peruvian fisheries data between 1971 and 1981, in thousands of tonnes (10^3 tons)

- anchoveta_stocks= [11019 4432 3982 5220 3954 5667 2272 2770 1506 1044 3407]
- merluza_stocks= [347 437 455 414 538 735 636 738 408 312 148]
- anchoveta_captures= [9184 3493 1313 3053 2673 3211 626 464 1000 223]
- merluza_captures= [26 13 133 109 85 93 107 303 93 159 69]



(a) Anchovy (b) Hake

Figure : Comparison of observed and simulated biomasses of anchovy and hake using a Lotka-Volterra model with density-dependence in the prey. Model parameters are $R = 2.25$, $L = 0.945$, $\kappa = 67\ 113\ \times 10^3$ t ($K = 37\ 285\ \times 10^3$ t), $\alpha = 1.22 \times 10^{-6}\ t^{-1}$, $\beta = 4.845 \times 10^{-8}\ t^{-1}$.

Here is the Lotka-Volterra *decision model*

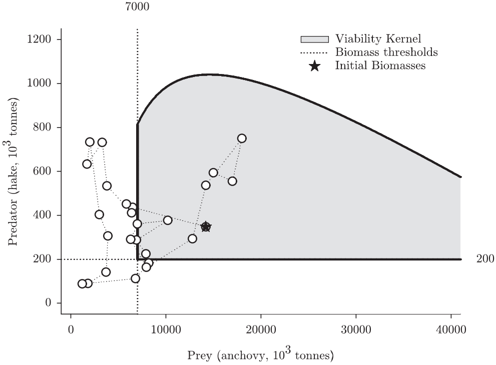
- A is the prey biomass (anchovy)
- H is the predator biomass (hake)
- The discrete-time Lotka-Volterra system is

$$
\begin{aligned}
A(t+1) &= A(t)\overbrace{\left(R - \frac{R}{\kappa}A(t) - \alpha H(t)\right)}^{\mathcal{R}_A\left(A(t),H(t)\right)}\left(1 - E_A(t)\right) \\
H(t+1) &= H(t)\underbrace{\left(L + \beta A(t)\right)}_{\mathcal{R}_H\left(A(t),H(t)\right)}\left(1 - E_H(t)\right),
\end{aligned}
$$

- The associated deterministic viability kernel is

$$
\mathbb{V}(t_0) = \left\{ \ (A,H) \mid A \geq S_A^\flat, \frac{1}{\alpha}[R - \frac{R}{\kappa}A - \frac{S_A^\flat + C_A^\flat}{A}] \geq H \geq \max\{\frac{S_H^\flat + C_H^\flat}{L+\beta A}, S_H^\flat\} \ \right\}
$$

For given biomasses and catches thresholds, we display the associated viability kernel



- Minimal biomasses thresholds
  - $S_A^\flat = 7\ 000\ kt$ (anchovy)
  - $S_H^\flat = 200\ kt$ (hake)
- Minimal catches thresholds
  - $C_A^\flat = 2\ 000\ kt$ (anchovy)
  - $C_H^\flat = 5\ kt$ (hake)

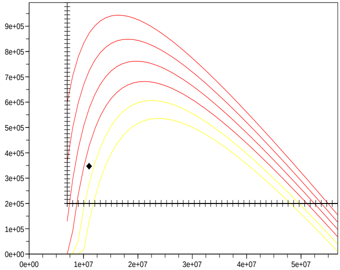
First acid test: plotting years of observed biomasses

- The range of values for viable states fits with measured biomasses
- Theoretically, a viable management with guaranteed biomasses and catches would have been possible since the initial state $\star$ is viable

Let us make a pause on our way towards ecosystem viable yields

- Let us turn back on what we have covered so far
  - taking in consideration both ecological and economic objectives
  - we have identified the viable states starting from which both objectives can be guaranteed as time flies
- And let us change the perspective
  - by first guaranteeing the ecological objectives
  - and then identifying compatible captures that can be guaranteed
  - when starting from a given initial state

We use the viability kernel the other way round, to design ecosystem viable yields



1. Considering that first are given minimal biomass conservation thresholds $S^\flat_A \geq 0$, $S^\flat_H \geq 0$
2. for initial biomasses $A_0 \geq S^\flat_A$ and $H_0 \geq S^\flat_H$, the following catch levels, if positive, can be sustainably maintained

$$C^{\flat,\star}_A(A_0, H_0) = \min\left\{S^\flat_A \mathcal{R}_A(S^\flat_A, S^\flat_H) - S^\flat_A ; A_0 \mathcal{R}_A(A_0, H_0) - S^\flat_A\right\}$$

$$C^{\flat,\star}_H(A_0, H_0) = \min\left\{S^\flat_H \mathcal{R}_H(S^\flat_A, S^\flat_H) - S^\flat_H ; H_0 \mathcal{R}_H(A_0, H_0) - S^\flat_H\right\}$$

And now, the second acid test... We compare theoretical ecosystem viable yields to Perú official quotas

	Viable yields (kt)		Perú official quotas (kt)	
	Model 1	Model 2	2006	2007
Anchovy	5 152	5 399		

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	Model 1	Model 2	2006	2007
Anchovy	5 152	5 399	4 250	5 300
Hake	49	56, 8	55	35

- Quotas are maximal bounds on catches
- Ecosystem viable yields (EVY) are minimal guaranteed yields
- EVY are obtained by "puzzling" viable effort rules:
  one can harvest more than the predator EVY to let the prey increase
- *Instituto del Mar del Perú* showed interest for this transparent method

Summary

- We have laid out examples of natural resources management problems where objectives are framed as constraints, using the apparatus of mathematical control theory
- We have provided solutions derived from viability theory methods
- And now, how do we move from deterministic dynamics and constraints to the uncertainty situation?

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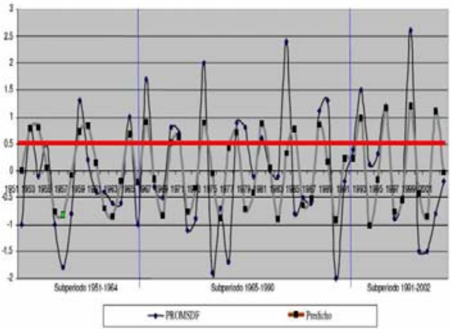
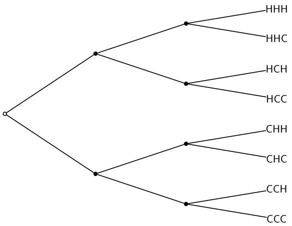




A control system connects input and output variables

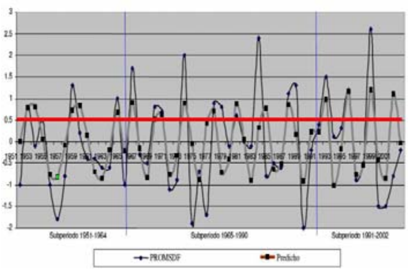
Input variables

Control	wood logs
Uncertainty	wood humidity
	metal conductivity

Output variables

soup quality
water vapor
temperature (internal state)

Uncertainty is pervasive in natural resources management



- Environmental uncertainties (*El Niño*)
- Habitats changes, mortality, natality
- Scientific uncertainties (structure of trophic networks, ecosystem services)

We plug incertain variables into the carbon cycle model "à la Nordhaus"

- Economic production $Q(t)$

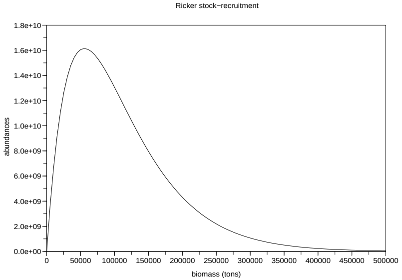
$$Q(t+1) = \Big(1 + \overbrace{g\big(w_e(t)\big)}^{\text{economic growth}}\Big) Q(t)$$

- CO_2 concentration $M(t)$

$$M(t+1) = M(t) - \delta(M(t) - M_{-\infty}) + \underbrace{\alpha(w_p(t))}_{\text{physics}} \overbrace{\mathtt{Emiss}\big(Q(t), w_z(t)\big)}^{\text{technologies}} \big(1 - a(t)\big)$$

- Vector of uncertainties $w(t) = (w_e(t), w_p(t), w_z(t))$ on
  - economic growth
  - technologies
  - climate dynamics

Uncertainties abound in population growth



- Stock-recruitment relationship condenses in one function complex mechanisms of birth, dispersion, predation, habitats, physical conditions, etc.
- Natural mortality (deseases, predation) between age-classes is poorly known

We plug incertain variables into the harvested age-class model

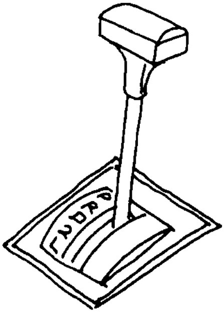
$$
\begin{aligned}
N_1(t+1) &= \mathtt{S/R}\Big(\mathtt{SSB}(N(t)), \underbrace{w(t)}_{\text{birth mortality, etc.}}\Big) \quad \text{recruitment} \\
N_2(t+1) &= e^{-(M_1+\lambda(t)F_1)} N_1(t) \\
\vdots &= \vdots \\
N_a(t+1) &= e^{-(\overbrace{M_{a-1}}^{\text{mortality}}+\lambda(t)F_{a-1})} N_{a-1}(t), \quad a = 2, \ldots, A-1 \\
N_A(t+1) &= e^{-(M_{A-1}+\lambda(t)F_{A-1})} N_{A-1}(t) + \pi e^{-(M_A+\lambda(t)F_A)} N_A(t)
\end{aligned}
$$

Uncertainty variables are new input variables

Input control variables are in the hands of the decision-maker at successive time periods

Control variables $u(t) \in \mathbb{U}$

The decision-maker can choose the values of control variables $u(t)$ at any time within given bounds



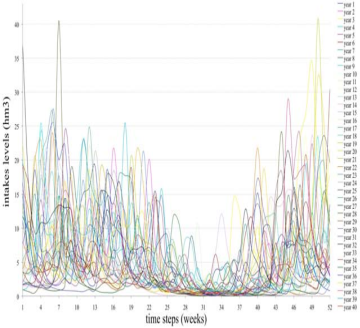
- at successive time periods
  - annual catches
  - years, months:
    starting of energy units like nuclear plants
  - weeks, days, intra-day: starting of hydropower units
- within given bounds
  - fishing quotas
  - turbined capacity

Input uncertain variables are out of the control of the decision-maker

Uncertain variables $w(t) \in \mathbb{W}$

are variables

- that take more than one single value (else they are deterministic)
- and over which the decision-maker (DM) has no control whatsoever



- Stationary parameters:
  unitary cost of CO_2 emissions
- Trends or seasonal effects:
  energy consumption pathway, mean temperatures, mean prices
- Stochastic processes:
  rain inputs in a dam, energy demand, prices
- Else (set membership):
  costs of climate change damage,
  water inflows in a dam

Uncertainty variables are new input variables in a discrete-time nonlinear state-control system

A specific output is distinguished, and is labeled state,
when the system may be written

$$x(t+1) = \mathtt{Dyn}\big(t, x(t), u(t), w(t)\big)\ , \quad t \in \mathbb{T} = \{t_0, t_0+1, \ldots, T-1\}$$

- time $t \in \overline{\mathbb{T}} = \{t_0, t_0+1, \ldots, T-1, T\} \subset \mathbb{N}$
  (the time period $[t, t+1[$ may be a year, a month, etc.)
- state $x(t) \in \mathbb{X} := \mathbb{R}^n$ (biomasses, abundances, etc.)
- control $u(t) \in \mathbb{U} := \mathbb{R}^p$ (catches or harvesting effort)
- uncertainty $w(t) \in \mathbb{W} := \mathbb{R}^q$
  (recruitment or mortality uncertainties, climate fluctuations or trends, etc.)
- dynamics $\mathtt{Dyn}$ maps $\mathbb{T} \times \mathbb{X} \times \mathbb{U} \times \mathbb{W}$ into $\mathbb{X}$
  (biomass model, age-class model, economic model)

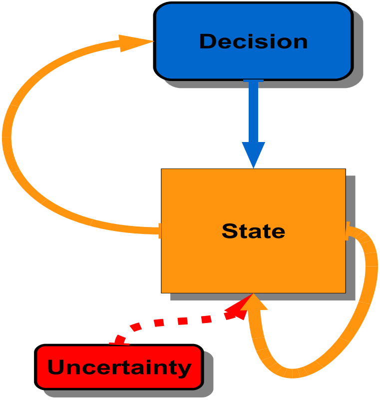
What have we covered so far?

Uncertainty variables are new input variables

$$x(t+1) = \mathtt{Dyn}\big(t, x(t), u(t), \underbrace{w(t)}_{\text{uncertainty}}\big)$$

- The future state $x(t+1)$ is no longer predictable
- because of the uncertain term $w(t)$,
- but the current state $x(t)$ carries information relevant for decision-making,
- and we shed light on the notion of policy

"Policies" are closed-loop controls



- Deterministic control theory appeals to open-loop control, ⌚ that is, a time-dependent sequence (planning, scheduling)

$$u : \underbrace{t \in \mathbb{T}}_{\text{time}} \mapsto \underbrace{u(t) \in \mathbb{U}}_{\text{control}}$$

- Another notion of solution is a decision rule, ⌚× 👁 a policy, that is, a mapping

$$\texttt{Pol} : \underbrace{(t,x) \in \mathbb{T} \times \mathbb{X}}_{\text{(time, state)}} \mapsto u = \underbrace{\texttt{Pol}(t,x) \in \mathbb{U}}_{\text{control}}$$

which "closes the loop" between time t–state x and control u (and is especially relevant in presence of uncertainties)

Summary

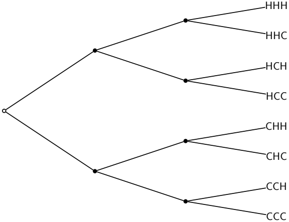
- Control variables are defined rather unambiguously:
  the DM can select their values at any time within given sets
- The distinction between input and output variables is relative to a system:
  for two interconnected dams, the water release from the upper to the lower dam can be "seen" as an input to the lower dam or as a control variable for the two-dams system
- In various examples of natural resources management,
  we have seen so-called uncertain variables
- Uncertain variables are variables
  - which take more than one single value (else they are deterministic)
  - and over which the decision-makers have no control whatsoever
- Uncertain and control variables combine in a dynamical model

We call scenario a temporal sequence of uncertainties

Scenarios are special cases of “states of Nature”

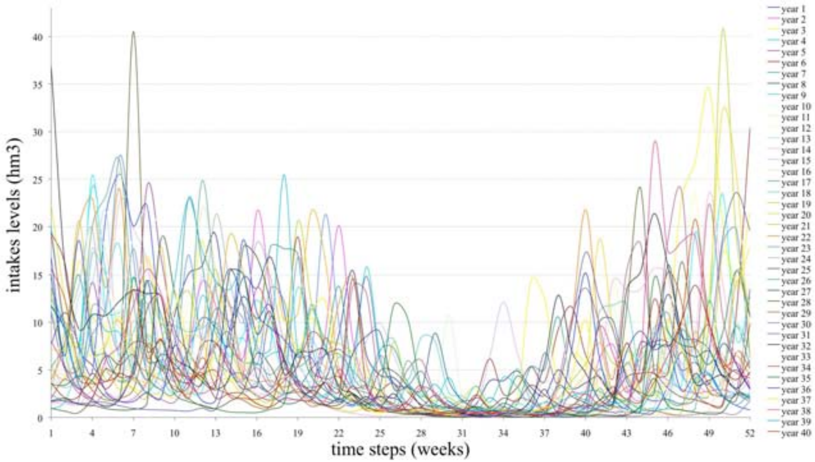
A scenario (pathway, chronicle) is a sequence of uncertainties

$$w(\cdot) := \big(w(t_0), \dots, w(T-1)\big) \in \Omega := \mathbb{W}^{T-t_0}$$

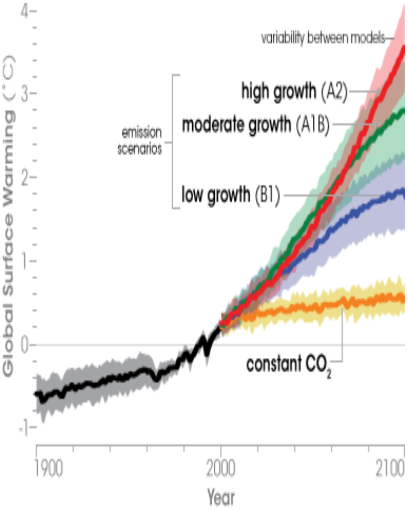


El tiempo se bifurca perpetuamente hacia innumerables futuros
(Jorge Luis Borges, *El jardín de senderos que se bifurcan*)

Water inflows historical scenarios



Beware! Scenario holds a different meaning in other scientific communities



- In practice, what modelers call a "scenario" is a mixture of
  - a sequence of uncertain variables (also called a pathway, a chronicle)
  - a policy `Pol`
  - and even a static or dynamical model
- In what follows

  scenario = pathway = chronicle

Along a given scenario, the system is deterministic

Une intelligence qui, à un instant donné, connaîtrait toutes les forces dont la nature est animée, la position respective des êtres qui la composent, si d'ailleurs elle était assez vaste pour soumettre ces données à l'analyse, embrasserait dans la même formule les mouvements des plus grands corps de l'univers, et ceux du plus léger atome. Rien ne serait incertain pour elle, et l'avenir comme le passé seraient présents à ses yeux.

Pierre-Simon Laplace,
Essai philosophique sur les probabilités

State and control solution maps are defined inductively along each scenario

Pick up

- a scenario $w(\cdot) = \big(w(t_0), w(t_0+1), \ldots, w(T)\big) \in \Omega$
- a policy $\texttt{Pol} : \underbrace{(t,x) \in \mathbb{T} \times \mathbb{X}}_{\text{(time, state)}} \mapsto u = \underbrace{\texttt{Pol}(t,x) \in \mathbb{U}}_{\text{control}}$
- an initial state $x(t_0) = x_0 \in \mathbb{X}$

1. Plug the state $x(t_0)$ into the "machine" Pol $\to$ initial decision $u(t_0) = \texttt{Pol}(t_0, x(t_0))$
2. Run the dynamics $\to$ second state $x(t_0+1) = \texttt{Dyn}(t_0, x(t_0), u(t_0), w(t_0))$
3. Second decision $u(t_0+1) = \texttt{Pol}(t_0+1, x(t_0+1))$
4. And so on $x(t_0+2) = \texttt{Dyn}(t_0+1, x(t_0+1), u(t_0+1), w(t_0+1))$
5. ...

State and control solution maps

Let be given

- a policy $\texttt{Pol} : \mathbb{T} \times \mathbb{X} \to \mathbb{U}$
- a scenario $w(\cdot) \in \Omega$
- and an initial state x_0 at initial time t_0

State solution map

The state solution map $X_{\texttt{Dyn}}[t_0, x_0, \texttt{Pol}, w(\cdot)]$ is the unique state path $x(\cdot) = (x(t_0), x(t_0+1), \ldots, x(T))$ solution of dynamic

$$x(t+1) = \texttt{Dyn}\big(t, x(t), \texttt{Pol}\big(t, x(t)\big), w(t)\big) , \quad t = t_0, \ldots, T-1$$

starting from the initial condition $x(t_0) = x_0$ at time t_0
and associated with policy $\texttt{Pol}$ and scenario $w(\cdot)$

The control solution map $U_{\texttt{Dyn}}[t_0, x_0, \texttt{Pol}, w(\cdot)]$ is the associated decision path $u(\cdot) = (u(t_0), u(t_0+1), \ldots, u(T-1))$ where $u(t) = \texttt{Pol}\big(t, x(t)\big)$
Everything above extends to the hazard-decision case

Summary

- A scenario is a temporal sequence of uncertainties
- State feedback policies correspond to perfect observation of the state
- State and control solution maps are defined inductively along each scenario
- Outputs of a state-control system with uncertainty are now contingent upon scenarios
- What is off-line information on scenarios?

A scenario is said to be viable for a given policy if the state and control trajectories satisfy the constraints

Viable scenario under given policy

A scenario $w(\cdot) \in \Omega$ is said to be viable under policy $\texttt{Pol} : \mathbb{T} \times \mathbb{X} \to \mathbb{U}$ if the trajectories $x(\cdot)$ and $u(\cdot)$ generated by the dynamics

$$x(t+1) = \texttt{Dyn}\big(t, x(t), u(t), w(t)\big) , \quad t = t_0, \ldots, T-1$$

with the policy

$$u(t) = \texttt{Pol}\big(t, x(t)\big)$$

satisfy the state and control constraints

$$\underbrace{u(t) \in \mathbb{B}\big(t, x(t)\big)}_{\text{control constraints}} \text{ and } \underbrace{x(t) \in \mathbb{A}(t)}_{\text{state constraints}} , \quad \forall t = t_0, \ldots, T$$

The set of viable scenarios is denoted by $\Omega_{\texttt{Pol}, t_0, x_0}$

We look after policies that make the corresponding set of viable scenarios "large"

Set of viable scenarios

$$\Omega_{\texttt{Pol},t_0,x_0} := \{ w(\cdot) \in \Omega \mid \begin{array}{l} \text{the state constraints} \\ X_{\texttt{Dyn}}[t_0, x_0, \texttt{Pol}, w(\cdot)](t) \in \mathbb{A}(t) \\ \text{and the control constraints} \\ U_{\texttt{Dyn}}[t_0, x_0, \texttt{Pol}, w(\cdot)] \in \mathbb{B}\big(t, x(t)\big) \\ \text{are satisfied for all times } t = t_0, \ldots, T \} \end{array}$$

- The larger set $\Omega_{\texttt{Pol},t_0,x_0}$ of viable scenarios, the better, because the policy Pol is able to maintain the system within constraints for a large "number" of scenarios
- But "large" in what sense? Robust? Probabilistic?

Outline of the presentation

1. Natural resources management issues and viability
   - Examples of decision models
   - Discrete–time viability
   - Are the ICES fishing quotas recommendations "sustainable"?
   - Ecosystem viable yields (anchovy–hake application)
2. Risk management, robust and stochastic viability
   - Uncertain systems, policies and viable scenarios
   - Robust viability
   - Robust viability kernel (anchovy–hake application)
   - Stochastic viability
   - Bycatches in a nephrops-hake fishery
   - Dam management under environmental/tourism constraint
3. Contribution to quantitative sustainable management

Robust viability dissects how to channel the system inside constraints whatever the scenarios

Let $\overline{\Omega} \subset \Omega$ be a subset of the set Ω of scenarios

The robust viability problem

Identify the initial states $x_0 \in \mathbb{X}$ for which there exists at least one viable robust policies $\texttt{Pol} : \mathbb{T} \times \mathbb{X} \to \mathbb{U}$ such that

- the state trajectories given by the state solution map $x(t) = X_{\texttt{Dyn}}[t_0, x_0, \texttt{Pol}, w(\cdot)](t)$ satisfy the following state constraints

$$x(t) \in \mathbb{A}(t) \text{ for } t = t_0, \ldots, T$$

- and the control constraints $u(t) = \texttt{Pol}\big(t, x(t)\big) \in \mathbb{B}\big(t, x(t)\big)$ are satisfied for $t = t_0, \ldots, T-1$

for all scenarios $w(\cdot) \in \overline{\Omega}$

The robust viability kernel is the set of initial states for which the robust viability problem can be solved

Robust viability kernel

$$\mathbb{V}\mathrm{iab}_1(t_0) := \left\{ x_0 \in \mathbb{X} \;\middle|\; \begin{array}{l} \text{there exists a policy } \mathtt{Pol} \in \mathfrak{U} \\ \text{ such that for all scenario } w(\cdot) \in \overline{\Omega} \\ \text{the state constraints } x(t) \in \mathbb{A}(t) \\ \text{and the control constraints} \\ u(t) = \mathtt{Pol}\big(t, x(t)\big) \in \mathbb{B}\big(t, x(t)\big) \\ \text{are satisfied for all times } t = t_0, \ldots, T \end{array} \right\}$$

where the state $x(t) = X_{\mathtt{Dyn}}[t_0, x_0, \mathtt{Pol}, w(\cdot)](t)$ is given by the state solution map

The robust viability kernel and viable scenarios are related

$$x_0 \in \underbrace{\mathbb{V}\mathrm{iab}_1(t_0)}_{\text{robust viability kernel}} \iff \left\{ \begin{array}{c} \text{there exists a policy } \mathtt{Pol} \in \mathfrak{U}, \\ \overline{\Omega} \subset \underbrace{\Omega_{\mathtt{Pol},t_0,x_0}}_{\text{viable scenarios}} \end{array} \right.$$

Robust viability kernels and robust viable policies can be defined for all times

Robust viability kernel at time t

The robust viability kernel at time t is the subset of states

$$\mathbb{V}\mathrm{iab}_1(t) := \left\{ x \in \mathbb{X} \;\middle|\; \begin{array}{l} \text{there exists } \mathtt{Pol} \in \mathfrak{U}^{ad} \text{ such that} \\ \text{for all scenario } w(\cdot) \in \overline{\Omega} \\ x(s) \in \mathbb{A}(s) \text{ for } s = t, \dots, T \end{array} \right\}$$

where $x(s) = X_{\mathtt{Dyn}}[t, x, \mathtt{Pol}, w(\cdot)](s)$ is given by the state solution map

The final viability kernel is the whole target set: $\mathbb{V}\mathrm{iab}_1(T) = \mathbb{A}(T)$

Viable robust policies

$$\mathfrak{U}_1^{\mathrm{viab}}(t, x) := \left\{ \mathtt{Pol} \in \mathfrak{U}^{ad} \;\middle|\; \begin{array}{l} \text{for all scenario } w(\cdot) \in \overline{\Omega} \\ X_{\mathtt{Dyn}}[t, x, \mathtt{Pol}, w(\cdot)](s) \in \mathbb{A}(s) \\ \text{for } s = t, \dots, T \end{array} \right\}$$

The viability program aims at turning state constraints into control constraints

- A priori constraints, with state constraints

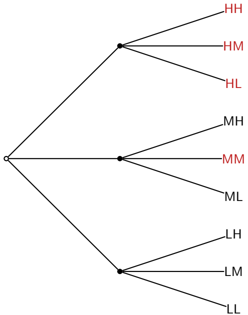
$$\begin{cases} x(t_0) \in \mathbb{X} \\ x(t+1) = \mathtt{Dyn}\big(t, x(t), u(t), w(t)\big) \\ u(t) \in \mathbb{B}\big(t, x(t)\big) & \text{control constraints} \\ x(t) \in \mathbb{A}(t) & \text{state constraints} \end{cases}$$

- are turned into a posteriori constraints, without state constraints except for the initial state

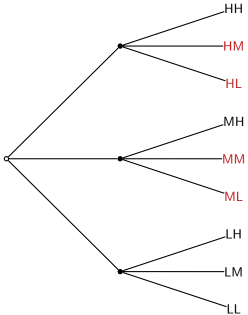
$$\begin{cases} x(t_0) \in \mathbb{V}\mathrm{iab}(t_0) & \text{initial state constraint} \\ x(t+1) = \mathtt{Dyn}\big(t, x(t), u(t), w(t)\big) \\ u(t) \in \mathbb{B}^{\mathrm{viab}}\big(t, x(t)\big) \subset \mathbb{B}\big(t, x(t)\big) & \text{control constraints} \end{cases}$$

- ex ante state constraints → ex post control constraints

Product scenarios subsets embody time independence



There is no time independence because the range of values of $w(t+1)$ depends on the value of $w(t)$:
$w(t) = H \Rightarrow w(t+1) \in \{M, L\}$
$w(t) = M \Rightarrow w(t+1) \in \{M\}$



There is time independence because
$\overline{\Omega} = \{H, M\} \times \{M, L\} \subset \Omega$
is a product set

A priori information on the scenarios may be set membership

The product case

- Uncertain variables may be restricted to subsets, period by period

$$w(t) \in \mathbb{S}(t)$$

so that some scenarios are selected and the rest are excluded

$$w(\cdot) \in \mathbb{S}(t_0) \times \cdots \times \mathbb{S}(T) \subset \Omega = \mathbb{W}^{T+1-t_0}$$

Bounded water inflows in a dam

If only an upper bound on water inflows is known, we represent off-line information by

$$0 \leq a(t) \leq a^\sharp$$

The robust dynamic programming equation is a backward equation relating the robust viability kernels

Let $\overline{\Omega} \subset \Omega$ be a subset of the set Ω of scenarios

Robust dynamic programming equation

If the scenarios vary within a rectangle $\overline{\Omega} = \mathbb{S}(t_0) \times \cdots \times \mathbb{S}(T)$ (corresponding to independence in the stochastic setting), the robust viability kernels satisfy the following backward induction, where t runs from $T-1$ down to t_0

$$\mathbb{V}\mathrm{iab}_1(T) = \mathbb{A}(T)$$

$$\mathbb{V}\mathrm{iab}_1(t) = \left\{ x \in \mathbb{A}(t) \;\middle|\; \begin{array}{l} \text{there exists an admissible control } u \in \mathbb{B}(t,x) \\ \text{such that for all scenarios } w \in \mathbb{S}(t) \\ \text{one has that } \mathtt{Dyn}(t,x,u,w) \in \mathbb{V}\mathrm{iab}_1(t+1) \end{array} \right\}$$

The robust dynamic programming equation yields the robust viable controls

Robust viable controls

For any time t and state x, robust viable controls are

$$\mathbb{B}_1^{\mathsf{viab}}(t,x) := \{u \in \mathbb{B}(t,x) \mid \forall w \in \mathbb{S}(t) \,,\, \mathtt{Dyn}(t,x,u,w) \in \mathbb{V}\mathrm{iab}_1(t+1)\}$$

Proposition

Viable robust policies are those $\mathtt{Pol} \in \mathfrak{U}$ *such that*

$$\mathtt{Pol}(t,x) \in \mathbb{B}_1^{viab}(t,x) \,, \quad \forall t \in \mathbb{T} \,, \forall x \in \mathbb{V}\mathrm{iab}_1(t)$$

The viability program is achieved

- Robust viable controls exist at time t if and only if the state x belongs to the robust viability kernel at time t:

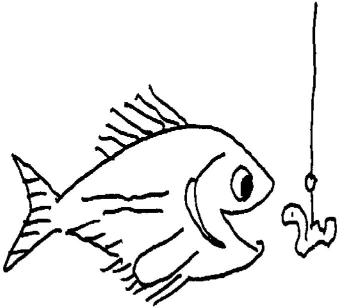
$$\mathbb{B}_1^{\text{viab}}(t,x) \neq \emptyset \iff x \in \mathbb{V}\mathrm{iab}_1(t)$$

- A solution to the viability problem is
  - an initial state x_0
  - and a policy Pol

  such that

$$\begin{aligned} x_0 &\in \mathbb{V}\mathrm{iab}_1(t_0) \\ \mathtt{Pol}(t,x) &\in \mathbb{B}_1^{\text{viab}}(t,x) , \quad \forall t \in \mathbb{T} , \forall x \in \mathbb{V}\mathrm{iab}_1(t) \end{aligned}$$

Outline of the presentation



We consider two species targeted by two fleets in a biomass ecosystem dynamics *with uncertainties*

We embody uncertainties, stocks and fishing interactions in a two-dimensional dynamical model

$$
\begin{aligned}
\overbrace{A(t+1)}^{\text{future biomass}} &= A(t)\overbrace{\mathcal{R}_A\big(A(t),H(t),\underbrace{w_A(t)}_{\text{uncer tainty}}\big)}^{\text{growth factor}}\big(1-\underbrace{E_A(t)}_{\text{effort control}}\big) \\
H(t+1) &= H(t)\mathcal{R}_H\big(A(t),H(t),\overbrace{w_H(t)}\big)\big(1-\overbrace{E_H(t)}\big)
\end{aligned}
$$

- Uncertainties $w_A(t)$ and $w_H(t)$ are discrepancies
- State vector $(A(t),H(t))$ represents biomasses
- Control vector $(E_A(t),E_H(t))$ is fishing effort of each species
- Catches are $E_A(t)\mathcal{R}_A\big(A(t),H(t),w_A(t)\big)A(t)$ and $E_H(t)\mathcal{R}_H\big(A(t),H(t),w_H(t)\big)H(t)$

Our objectives are twofold: conservation and production

The robust viability kernel is the set of initial species biomasses $\big(A(t_0), H(t_0)\big)$
from which at least one appropriate policy
produces biomasses and effort trajectories
such that the following goals are satisfied
for all the scenarios $\big(w_A(t), w_H(t)\big)$, $t = t_0, t_0 + 1, \ldots, T$

- preservation (minimal biomass thresholds)

$$A \text{ \underline{stocks}}: \qquad A(t) \geq S_A^\flat$$

$$H \text{ \underline{stocks}}: \qquad H(t) \geq S_H^\flat$$

- economic/social requirements (minimal catch thresholds)

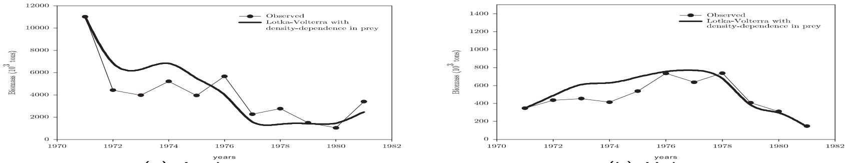
$$A \text{ \underline{catches}}: \qquad E_A(t)\mathcal{R}_A\big(A(t), H(t), w_A(t)\big)A(t) \geq C_A^\flat$$

$$H \text{ \underline{catches}}: \qquad E_H(t)\mathcal{R}_H\big(A(t), H(t), w_H(t)\big)H(t) \geq C_H^\flat$$

We taylor a Lotka-Volterra *decision model* to hake-anchovy Peruvian fisheries scarce data, and qualify the discrepancies as uncertainties

Hake-anchovy Peruvian fisheries data between 1971 and 1981, in thousands of tonnes (10^3 tons)

- anchoveta_stocks= [11019 4432 3982 5220 3954 5667 2272 2770 1506 1044 3407]
- merluza_stocks= [347 437 455 414 538 735 636 738 408 312 148]
- anchoveta_captures= [9184 3493 1313 3053 2673 3211 626 464 1000 223]
- merluza_captures= [26 13 133 109 85 93 107 303 93 159 69]



(a) Anchovy (b) Hake

Figure : Comparison of observed and simulated biomasses of anchovy and hake using a Lotka-Volterra model with density-dependence in the prey. Model parameters are $R = 2.25$, $L = 0.945$, $\kappa = 67\,113 \times 10^3$ t ($K = 37\,285 \times 10^3$ t), $\alpha = 1.22 \times 10^{-6}\ t^{-1}$, $\beta = 4.845 \times 10^{-8}\ t^{-1}$.

Here is the Lotka-Volterra *decision model* with uncertainty

- A is the prey biomass (anchovy)
- H is the predator biomass (hake)
- The discrete-time Lotka-Volterra system with uncertainty is

$$
\begin{aligned}
A(t+1) &= A(t) \overbrace{\left(w_A(t) + R - \frac{R}{\kappa} A(t) - \alpha H(t)\right)}^{\mathcal{R}_A\left(A(t), H(t), w_A(t)\right)} \left(1 - E_A(t)\right) \\
H(t+1) &= H(t) \underbrace{\left(w_H(t) + L + \beta A(t)\right)}_{\mathcal{R}_H\left(A(t), H(t), w_H(t)\right)} \left(1 - E_H(t)\right),
\end{aligned}
$$

We make a tough assumption on the set of scenarios

- An uncertainty scenario is a time sequence of uncertainty couples

$$\big(w_A(\cdot), w_H(\cdot)\big) = \Big(\big(w_A(t_0), w_H(t_0)\big), \ldots, \big(w_A(T-1), w_H(T-1)\big) \Big)$$

- We assume that, at each time t,
  the uncertainties $\big(w_A(t), w_H(t)\big)$ can take any value in a two-dimensional set

$$\big(w_A(t), w_H(t)\big) \in \mathbb{S}(t) \subset \mathbb{R}^2$$

- Therefore, from one time t to the next $t+1$, uncertainties can be drastically different, since $\big(w_A(t), w_H(t)\big)$ is not related to $\big(w_A(t+1), w_H(t+1)\big)$
- Such an independence assumption is materialized by the property that
  a scenario can take any value in a product set

$$\big(w_A(\cdot), w_H(\cdot)\big) \in \prod_{t=t_0}^{T-1} \mathbb{S}(t)$$

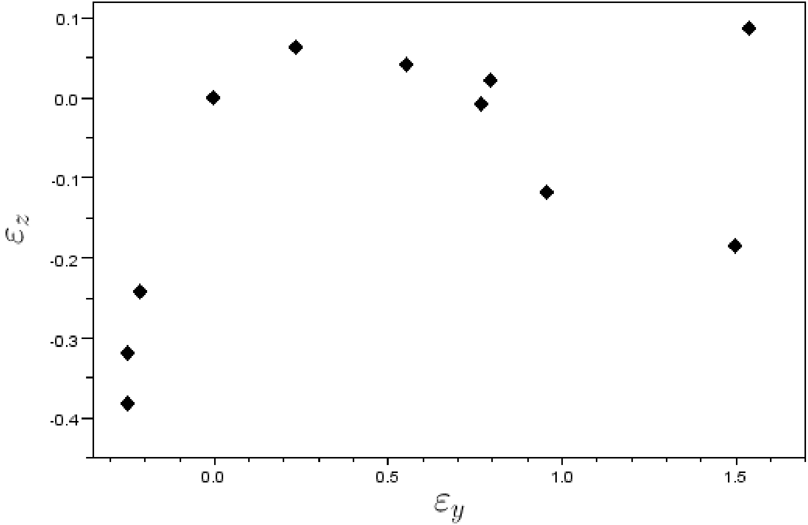
In practice, we consider stationary uncertainty sets forged from empirical data

- In practice, we consider stationary uncertainty sets $\mathbb{S}(t) = \mathbb{S}$
- We define $\overline{w}_A(t)$ and $\overline{w}_H(t)$ such that

$$\begin{cases} \overline{A}(t+1) &= \overline{A}(t)\big(\overline{w}_A(t) + R - \frac{R}{\kappa}\overline{A}(t) - \alpha\overline{H}(t)\big)\big(1 - \overline{v}_A(t)\big) \\ \overline{H}(t+1) &= \overline{H}(t)\big(\overline{w}_H(t) + L + \beta\overline{A}(t)\big)\big(1 - \overline{v}_H(t)\big) \end{cases}$$

  where $(\overline{A}(t), \overline{H}(t))_{t=t_0,\ldots,T}$ and $(\overline{v}_A(t), \overline{v}_H(t))_{t=t_0,\ldots,T-1}$ denote the empirical biomass and effort trajectories
- Therefore, our tough assumption on the set of scenarios is:
  any of the possible uncertainty of any year
  can materialize any *other* year

Empirical distribution of the uncertainties $(\overline{w}_A(t), \overline{w}_H(t))_{t=t_0,\dots,T-1}$



We first consider the empirical uncertainty set and a refinement

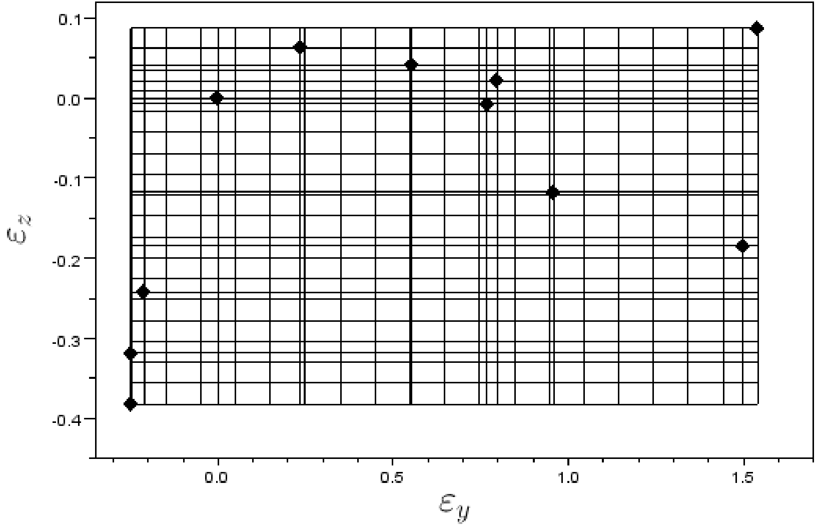
- The empirical uncertainties set is

$$\mathbb{S}^E = \underbrace{\{(\overline{w}_A(t), \overline{w}_H(t)) | t = t_0, \ldots, T-1\}}_{\text{empirical discrepancies}} \cup \underbrace{\{(0,0)\}}_{\text{deterministic case}}$$

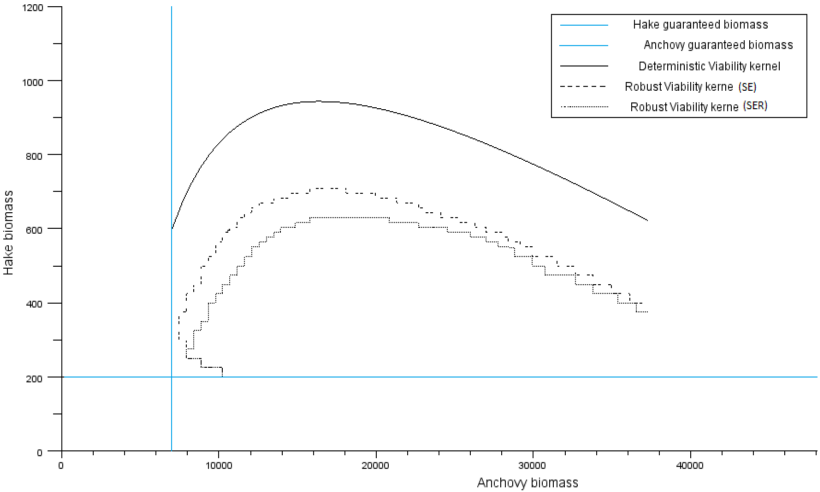
- The refined empirical uncertainties set $\mathbb{S}^{ER}$
is made of 900 uncertainty couples delineated by a 30×30 grid over the surface $[\overline{w}_A^{min}, \overline{w}_A^{max}] \times [\overline{w}_H^{min}, \overline{w}_H^{max}]$, including all the uncertainty couples in $\mathbb{S}^E$
- Since $\{(0,0)\} \subset \mathbb{S}^E \subset \mathbb{S}^{ER}$,
the corresponding robust and deterministic viability kernels satisfy

$$\mathbb{V}\mathrm{iab}_1^{ER}(t_0) \subset \mathbb{V}\mathrm{iab}_1^{E}(t_0) \subset \mathbb{V}\mathrm{iab}(t_0)$$

Figure : Uncertainty sets $\mathbb{S}^E$ (diamonds) and $\mathbb{S}^{ER}$ (grid)



The robust viability kernels are noticeably smaller than the deterministic one



Now, we focus on worst-case uncertainties

- Numerical simulations led us to consider the three following uncertainty sets
  - $$\mathbb{S}^L = \{(\frac{\overline{w}_A^{min}}{2}, \frac{\overline{w}_H^{min}}{2}), (\frac{\overline{w}_A^{min}}{2}, \frac{\overline{w}_H^{max}}{2})\}$$
  - $$\mathbb{S}^M = \{(\overline{w}_A^{min}, \overline{w}_H^{min}), (\overline{w}_A^{min}, \overline{w}_H^{max})\}$$
  - $$\mathbb{S}^H = 1.1 * \mathbb{S}^M$$
- Since $\{(0,0)\} \subset \mathbb{S}^L \subset \mathbb{S}^M \subset \mathbb{S}^H$, the corresponding robust and deterministic viability kernels satisfy

$$\mathbb{V}\mathrm{iab}_1^H(t_0) \subset \mathbb{V}\mathrm{iab}_1^M(t_0) \subset \mathbb{V}\mathrm{iab}_1^L(t_0) \subset \mathbb{V}\mathrm{iab}(t_0)$$

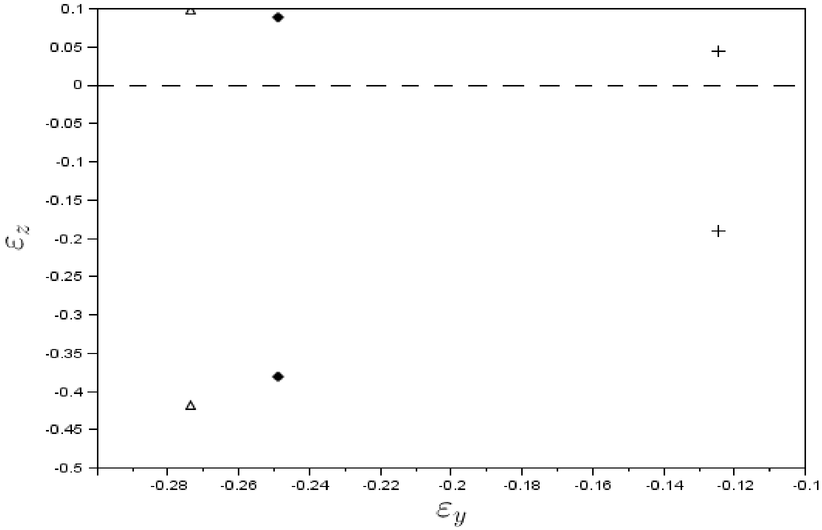
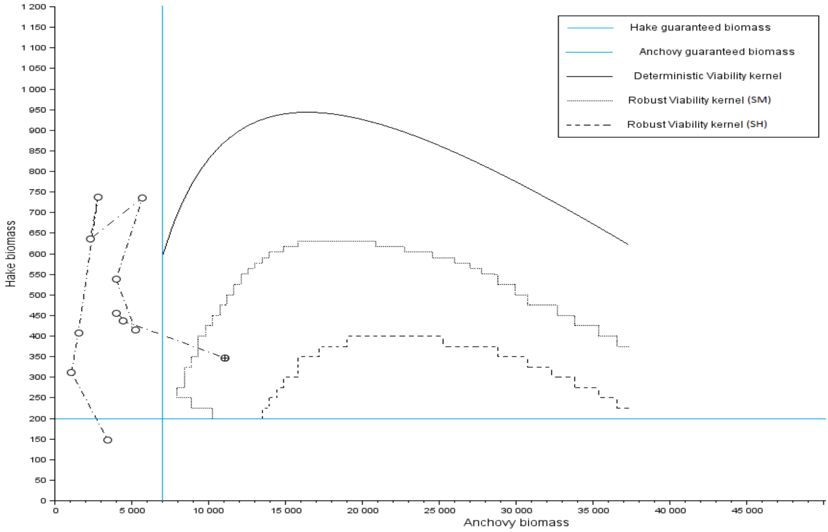
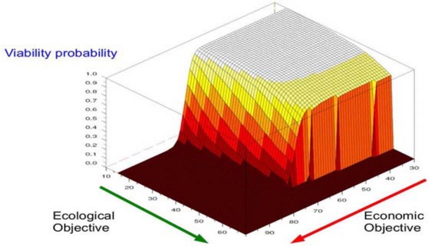


Figure : Uncertainty sets $\mathbb{S}^L$ (crosses), $\mathbb{S}^M$ (diamonds) and $\mathbb{S}^H$ (triangles)

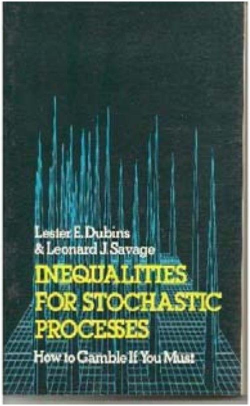
Figure : Robust viability kernels $\mathbb{V}\text{iab}_1^L(t_0)$, $\mathbb{V}\text{iab}_1^M(t_0)$ and $\mathbb{V}\text{iab}_1^H(t_0)$ and the deterministic viability kernel



Outline of the presentation



Maximizing the probability of success may be an objective



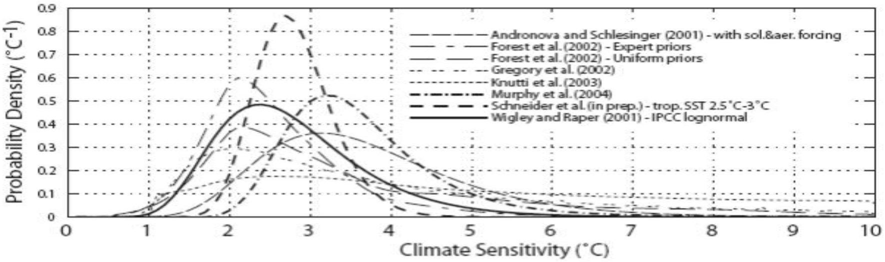
How to gamble if you must, L.E. Dubbins and L.J. Savage, 1965

Imagine yourself at a casino with $1,000. For some reason, you desperately need $10,000 by morning; anything less is worth nothing for your purpose.

The only thing possible is to gamble away your last cent, if need be, in an attempt to reach the target sum of $10,000.

- The question is how to play, not whether. What ought you do? How should you play?
  - Diversify, by playing 1 $ at a time?
  - Play boldly and concentrate, by playing 10,000 $ only one time?
- What is your decision criterion?

The set Ω of scenarios can be equipped with a probability $\mathbb{P}$ (though this is a delicate issue!)



In practice, one often assumes that the components $\big(w(t_0), \ldots, w(T-1)\big)$ form an independent and identically distributed sequence of random variables, or form a Markov chain, or a time series

The viability probability is the probability of satisfying constraints under a policy

Viability probability

The viability probability associated with
the initial time t_0, the initial state x_0 and the policy $\texttt{Pol}$
is the probability $\mathbb{P}\left[\Omega_{\texttt{Pol},t_0,x_0}\right]$ of the set $\Omega_{\texttt{Pol},t_0,x_0}$ of viable scenarios

$$\mathbb{P}\left[\Omega_{\texttt{Pol},t_0,x_0}\right] = \text{Proba}\left\{\begin{array}{l}\text{scenarios along which}\\ \text{the state } x(\cdot) \text{ and control } u(\cdot) \text{ trajectories}\\ \text{generated by dynamics } \texttt{Dyn} \text{ and policy } \texttt{Pol}\\ \text{starting from initial state } x_0 \text{ at initial time } t_0\\ \text{satisfy the constraints}\\ \text{from initial time } t_0 \text{ to horizon } T\end{array}\right\}$$

The maximal viability probability is the upper bound for the probability of satisfying constraints

Maximal viability probability and optimal viable policy

The maximal viability probability is

$$\max_{\texttt{Pol}} \mathbb{P}\left[\Omega_{\texttt{Pol},t_0,x_0}\right]$$

An optimal viable policy $\texttt{Pol}^\star$ satisfies

$$\mathbb{P}\left[\Omega_{\texttt{Pol}^\star,t_0,x_0}\right] \geq \mathbb{P}\left[\Omega_{\texttt{Pol},t_0,x_0}\right]$$

In a sense, any optimal viable policy makes the set of viable scenarios the "largest" possible

Let us introduce the stochastic viability Bellman function

Suppose that the primitive random variables
$\big(w(t_0), w(t_0+1), \dots, w(T-2), w(T-1)\big)$
are independent under the probability $\mathbb{P}$

Bellman function / stochastic viability value function

Define the probability-to-go as

$$V(t,x) :=$$

$$\max_{\texttt{Pol}} \mathbb{P}\Big(w(\cdot) \in \Omega \mid \overbrace{\texttt{Pol}\big(s, x(s)\big) \in \mathbb{B}\big(s, x(s)\big)}^{\text{control constraints}} \text{ and } \overbrace{x(s) \in \mathbb{A}(s)}^{\text{state constraints}} \text{ for } s \geq t \Big)$$

where $x(s+1) = \texttt{Dyn}\big(s, x(s), \texttt{Pol}\big(s, x(s)\big), w(s)\big)$ and $x(t) = x$

- The function $V(t,x)$ is called stochastic viability value function or Bellman function
- The original problem is $V(t_0, x_0)$

The dynamic programming equation is a backward equation satisfied by the stochastic viability value function

Proposition

If the primitive random variables $\big(w(t_0), w(t_0+1), \ldots, w(T-2), w(T-1)\big)$ *are independent under the probability* $\mathbb{P}$, *the stochastic viability value function* $V(t,x)$ *satisfies the following backward induction, where* t *runs from* $T-1$ *down to* t_0

$$
\begin{aligned}
V(T,x) &= \mathbf{1}_{\mathbb{A}(T)}(x) \\
V(t,x) &= \mathbf{1}_{\mathbb{A}(t)}(x) \max_{u \in \mathbb{B}(t,x)} \mathbb{E}_{w(t)} \Big[V\Big(t+1, \mathtt{Dyn}(t,x,u,w(t)) \Big) \Big]
\end{aligned}
$$

The stochastic viable dynamic programming equation yields stochastic viable policies

For any time t and state x, let us assume that the set

$$\mathbb{B}^{\mathsf{viab}}(t,x) := \operatorname*{argmax}_{u \in \mathbb{B}(t,x)} \Big(\mathbf{1}_{\mathbb{A}(t)}(x) \mathbb{E}_{w(t)} \Big[V\Big(t+1, \mathtt{Dyn}(t,x,u,w(t)) \Big) \Big] \Big)$$

of viable controls is not empty

Proposition

Then, any (measurable) policy $\mathtt{Pol}$ *such that* $\mathtt{Pol}^{\star}(t,x) \in \mathbb{B}^{viab}(t,x)$ *is an optimal viable policy which achieves the maximal viability probability*

$$V(t_0,x_0) = \max_{\mathtt{Pol}} \mathbb{P}\left[\Omega_{\mathtt{Pol},t_0,x_0}\right]$$

Outline of the presentation

We set up a dynamical age-class model of hake and nephrops in technical interaction

$$N_1^h(t+1) = w^h(t) \text{ uncertain hake recruitment}$$
$$N_1^n(t+1) = w^n(t) \text{ uncertain nephrops recruitment}$$

$$N_a^h(t+1) = N_{a-1}^h(t)\left(1 - M_{a-1}^h - \overbrace{u(t)F_{a-1}^{nh}}^{\text{hake bycatch}} - F_{a-1}^{hh}\right)$$

$$N_a^n(t+1) = N_{a-1}^n(t)\left(1 - M_{a-1}^n - \overbrace{u(t)F_{a-1}^{nn}}^{\text{nephrops fishing mortality}}\right)$$

$$N_A^h(t+1) = N_{A-1}^h(t)\left(1 - M_{A-1}^h - u(t)F_{A-1}^{nh} - F_{A-1}^{hh}\right) + N_A^h(t)\left(1 - M_A^h - u(t)F_A^{nh} - F_A^{hh}\right)$$

$$N_A^n(t+1) = N_{A-1}^n(t)\left(1 - M_{A-1}^n - u(t)F_{A-1}^{nn}\right) + N_A^n(t)\left(1 - M_A^n - u(t)F_A^{nn}\right)$$

The relative effort of the nephrops fleet has to be controlled to ensure both nephrops fleet profitability and hake preservation

- Economic objective: nephrops fishery is economically viable if the gross return is greater than a threshold

$$\underbrace{P\big(N^n(t), u(t)\big)}_{\text{payoff}} \geq P^\flat$$

- Ecological objective: fishery is ecologically viable if its impact by bycatch on the hake biology is compatible with sufficient recruitment of mature hakes

$$\underbrace{N_4^h(t)}_{\text{fourth age-class}} \geq (N_4^h)^\flat$$

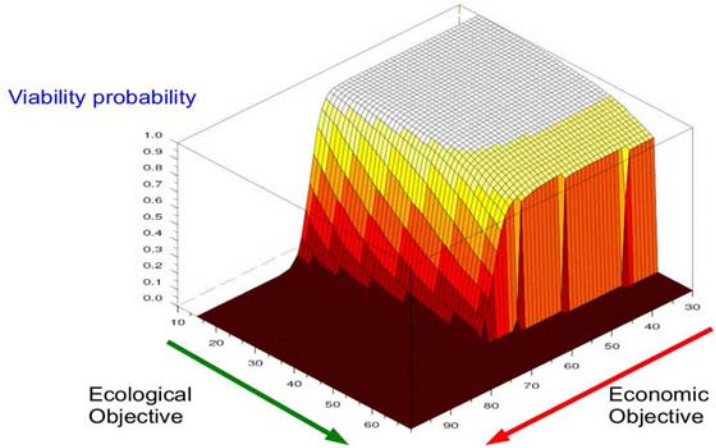
An optimal viable policy can be calculated thanks to monotonicity properties

- Due to monotonicity properties
  - of the dynamics, increasing in the state variable and decreasing in the control
  - of the constraints, increasing in the state variable and decreasing in the control
- we can prove that

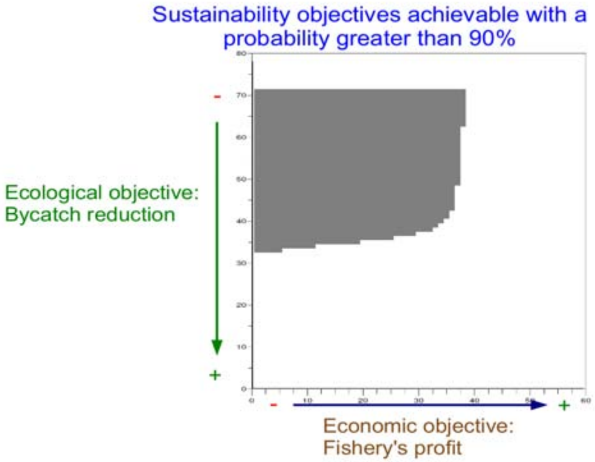
$$\mathtt{Pol}^{\star}(t,N) = \inf\{u \in [0,u^{\sharp}] \mid P(N^{n},u) \geq P^{\flat}\}$$

is an optimal viable policy

We draw the maximal viability probability as a function of the guaranteed thresholds $P^\flat$ and $(N_4^h)^\flat$



We draw the iso-values for the maximal viability probability as a function of guaranteed thresholds $P^\flat$ and $(N_4^h)^\flat$

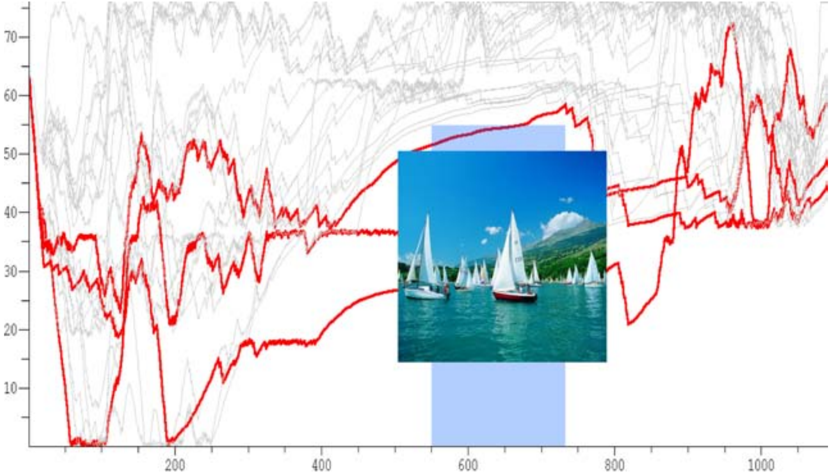


Outline of the presentation

Tourism issues impose constraints upon traditional economic management of a hydro-electric dam

- Maximizing the revenue from turbinated water
- under a tourism constraint of having enough water in July and August

The red stock trajectories fail to meet the tourism constraint in July and August



We consider a single dam nonlinear dynamical model in the decision-hazard setting

We can model the dynamics of the water volume in a dam by

$$\underbrace{S(t+1)}_{\text{future volume}} = \min\{S^\sharp, \underbrace{S(t)}_{\text{volume}} - \underbrace{q(t)}_{\text{turbined}} + \underbrace{a(t)}_{\text{inflow volume}}\}$$

- $S(t)$ volume (stock) of water at the beginning of period $[t, t+1[$
- $a(t)$ inflow water volume (rain, etc.) during $[t, t+1[$
- $q(t)$ turbined outflow volume during $[t, t+1[$
  - decided at the beginning of period $[t, t+1[$
  - chosen such that $0 \leq q(t) \leq \min\{S(t), q^\sharp\}$
  - supposed to depend on the stock $S(t)$ but not on the inflow water $a(t)$
- the setting is called decision-hazard:
  $a(t)$ is not available at the beginning of period $[t, t+1[$

In the risk-neutral economic approach, an optimal management maximizes the expected payoff

- Suppose that
  - at the horizon, the final volume $S(T)$ has a value $\texttt{UtilFin}\big(S(T)\big)$, the "final value of water"
  - turbined water $q(t)$ is sold at price $p(t)$, related to the price at which energy can be sold at time t
  - a probability $\mathbb{P}$ is given on the set $\Omega = \mathbb{R}^{T-t_0} \times \mathbb{R}^{T-t_0}$ of water inflows scenarios $\big(a(t_0), \ldots, a(T-1)\big)$ and prices scenarios $\big(p(t_0), \ldots, p(T-1)\big)$
- The traditional (risk-neutral) economic problem is to maximize the intertemporal payoff (without discounting if the horizon is short)

$$\max \mathbb{E}\left[\sum_{t=t_0}^{T-1} \overbrace{p(t)}^{\text{price}} \underbrace{q(t)}_{\text{turbined}} - \epsilon q(t)^2 + \overbrace{\texttt{UtilFin}\big(S(T)\big)}^{\text{final volume utility}}\right]$$

We now have a stochastic optimization problem, where the tourism constraint still needs to be dressed in formal clothes

- Traditional cost minimization/payoff maximization

$$\max \mathbb{E}\left[\sum_{t=t_0}^{T-1} \overbrace{p(t)q(t) - \epsilon q(t)^2}^{\text{turbined water payoff}} + \overbrace{\mathtt{UtilFin}(S(T))}^{\text{final volume utility}}\right]$$

- Tourism constraint

$$\text{volume} \quad S(t) \geq S^\flat \ , \quad \forall t \in \{\text{July, August}\}$$

- In what sense should we consider this inequality which involves the random variables $S(t)$ for $t \in \{\text{July, August}\}$?

Robust / almost sure / probability constraint

- Robust constraints: for all the scenarios in a subset $\overline{\Omega} \subset \Omega$

$$S(t) \geq S^{\flat} \; , \quad \forall t \in \{ \text{July}, \text{August} \}$$

- Almost sure constraints

$$\text{Probability} \left\{ \begin{array}{l} \text{water inflow scenarios along which} \\ \text{the volumes } S(t) \text{ are above the} \\ \text{threshold } S^{\flat} \text{ for periods } t \text{ in summer} \end{array} \right\} = 1$$

- Probability constraints, with “confidence” level $p \in [0,1]$

$$\text{Probability} \left\{ \begin{array}{l} \text{water inflow scenarios along which} \\ \text{the volumes } S(t) \text{ are above the} \\ \text{threshold } S^{\flat} \text{ for periods } t \text{ in summer} \end{array} \right\} \geq p$$

- and also by penalization, or in the mean, etc.

Our problem may be clothed as a stochastic optimization problem under a probability constraint

- The traditional economic problem is $\max \mathbb{E}[P(T)]$ where the payoff/utility criterion is

$$P(T) = \sum_{t=t_0}^{T-1} \overbrace{p(t)q(t) - \epsilon q(t)^2}^{\text{turbined water payoff}} + \overbrace{\texttt{UtilFin}(S(T))}^{\text{final volume utility}}$$

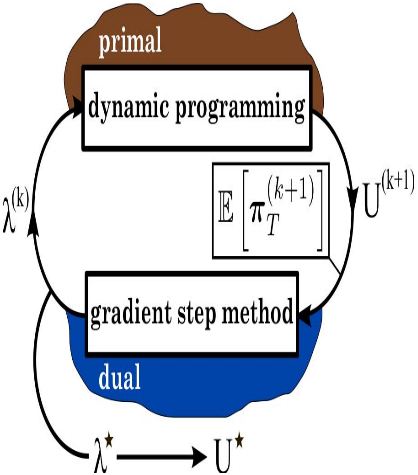
- and a failure tolerance is accepted

$$\text{Probability} \left\{ \begin{array}{l} \text{water inflow scenarios along which} \\ \text{the volumes } S(t) \geq S^\flat \\ \text{for periods } t \text{ in July and August} \end{array} \right\} \geq 90\%$$

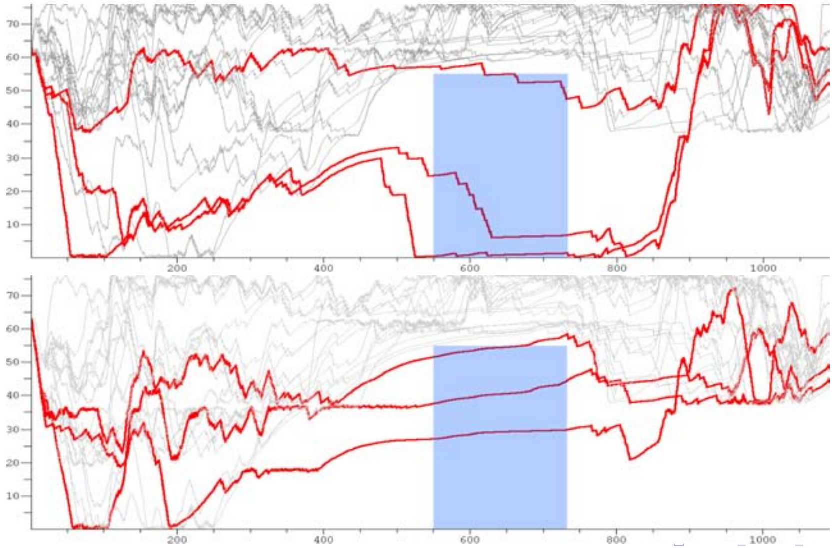
- Details concerning the theoretical and numerical resolution are available on demand ;-)

Details concerning the theoretical and numerical resolution are available on demand ;-)

- $\boldsymbol{\pi}_0 = 1$ and $\boldsymbol{\pi}_{t+1} = \begin{cases} \mathbf{1}_{\{x_{t+1} \geq x_{\text{ref}}\}} \times \boldsymbol{\pi}_t & \text{if } t \in \mathcal{T} \\ \boldsymbol{\pi}_t & \text{else} \end{cases}$
- $\mathbb{P}\left[x_\tau \geq x_{\text{ref}} \,,\ \ \forall \tau \in \mathcal{T}\right] = \mathbb{E}\left[\mathbf{1}_{\{x_\tau \geq x_{\text{ref}} \,,\ \forall \tau \in \mathcal{T}\}}\right] = \mathbb{E}\left[\prod_{\tau \in \mathcal{T}} \mathbf{1}_{\{x_\tau \geq x_{\text{ref}}\}}\right] = \mathbb{E}\left[\boldsymbol{\pi}_T\right]$



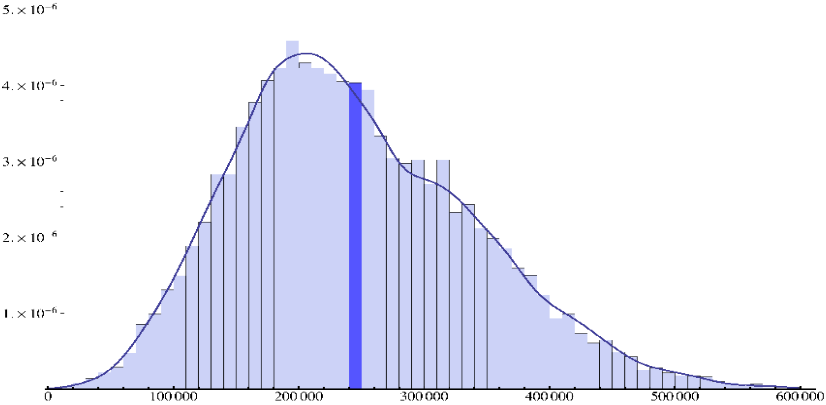
90% of the stock trajectories meet the tourism constraint in July and August



Our resolution approach brings a sensible improvement compared to standard procedures

OPTIMAL POLICIES	OPTIMIZATION		SIMULATION		
	Iterations	Time	Gain	Respect	Well behaviour
Standard	15	10 mn	**ref**	**0,9**	**no**
Convenient	**10**	**160 mn**	**-3.20%**	**0,9**	**yes**
Heuristic	10	160 mn	**-3.25%**	**0,9**	**yes**

However, though the expected payoff is optimal, the payoff effectively realized can be far from it



We propose a stochastic viability formulation to treat symmetrically and to guarantee both environmental and economic objectives

- Given two thresholds to be guaranteed
  - a volume $S^\flat$ (measured in cubic hectometers hm^3)
  - a payoff $P^\flat$ (measured in numeraire \$)
- we look after policies achieving the maximal viability probability

$$\Pi(S^\flat, P^\flat) = \max \text{Proba} \left\{ \begin{array}{l} \text{water inflow scenarios along which} \\ \text{the volumes } S(t) \geq S^\flat \\ \text{for all time } t \in \{\text{July, August}\} \\ \text{and the final payoff } P(T) \geq P^\flat \end{array} \right\}$$

- $\Pi(S^\flat, P^\flat)$ is the maximal probability to guarantee to be above the thresholds $S^\flat$ and $P^\flat$

The stochastic viability formulation requires to redefine state and dynamics

- The state is the couple $x(t) = \big(S(t), P(t)\big)$ volume/payoff
- The control $u(t) = q(t)$ is the turbined water
- The dynamics is

$$\underbrace{S(t+1)}_{\text{future volume}} = \min\{S^\sharp, \underbrace{S(t)}_{\text{volume}} - \underbrace{q(t)}_{\text{turbined}} + \underbrace{a(t)}_{\text{inflow volume}} \}, \quad t = t_0, \ldots, T-1$$

$$\underbrace{P(t+1)}_{\text{future payoff}} = \underbrace{P(t)}_{\text{payoff}} + \underbrace{p(t)q(t) - \epsilon q(t)^2}_{\text{turbined water payoff}}, \quad t = t_0, \ldots, T-2$$

$$P(T) = P(T-1) + \underbrace{\texttt{UtilFin}\big(S(T)\big)}_{\text{final volume utility}}$$

In the stochastic viability formulation, objectives are formulated as state constraints

- The control constraints are

$$u(t) \in \mathbb{B}\big(t, x(t)\big) \iff 0 \leq q(t) \leq S(t)$$

- The state constraints are

$$x(t) \in \mathbb{A}(t) \iff \begin{cases} S(t) \geq S^{\flat} & , \quad \forall t \in \{\text{July, August}\} \\ P(T) \geq P^{\flat} & \end{cases}$$

For each couple of thresholds on payoff and stock, we write a dynamic programming equation

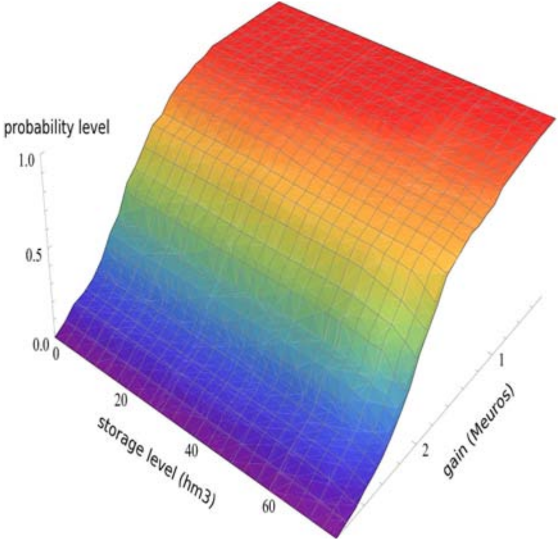
- Abstract version

$$
\begin{aligned}
V(T,x) &= \mathbf{1}_{\mathbb{A}(T)}(x) \\
V(t,x) &= \mathbf{1}_{\mathbb{A}(t)}(x) \max_{u \in \mathbb{B}(t,x)} \mathbb{E}_{w(t)} \Big[V\Big(t+1, \mathtt{Dyn}\big(t,x,u,w(t)\big) \Big) \Big]
\end{aligned}
$$

- Specific version

$$
\begin{aligned}
V(T,S,P) &= \mathbf{1}_{\{P \geq P^\flat\}} \\
V(T-1,S,P) &= \max_{0 \leq q \leq S} \mathbb{E}_{a(t)} \Big[V\Big(t+1, S-q+a(t), P+\mathtt{UtilFin}(S) \Big) \Big] \\
V(t,S,P) &= \max_{0 \leq q \leq S} \mathbb{E}_{a(t)} \Big[V\Big(t+1, S-q+a(t), P+pq-\epsilon q^2 \Big) \Big] \;, \; t \notin \{\,\text{July, August}\,\} \\
V(t,S,P) &= \mathbf{1}_{\{S \geq S^\flat\}} \max_{0 \leq q \leq S} \mathbb{E}_{a(t)} \Big[V\Big(t+1, S-q+a(t), P+pq-\epsilon q^2 \Big) \Big] \;, \\
& \quad t \in \{\,\text{July, August}\,\}
\end{aligned}
$$

We plot the maximal viability probability $\Pi(S^\flat, P^\flat)$ as a function of guaranteed thresholds $S^\flat$ and $P^\flat$

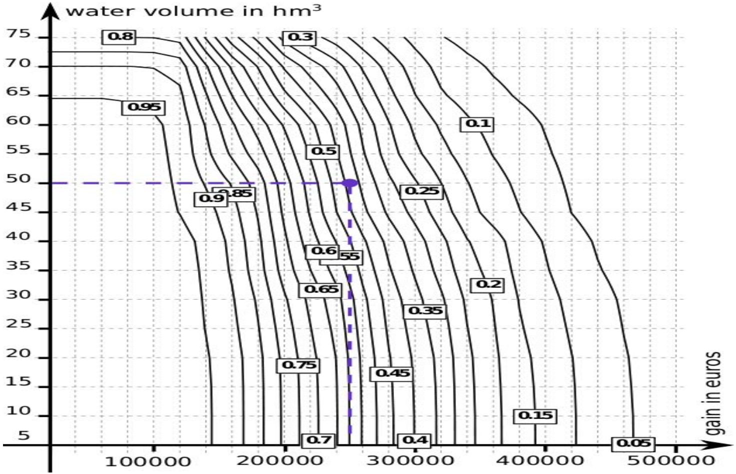


For example, the probability to guarantee

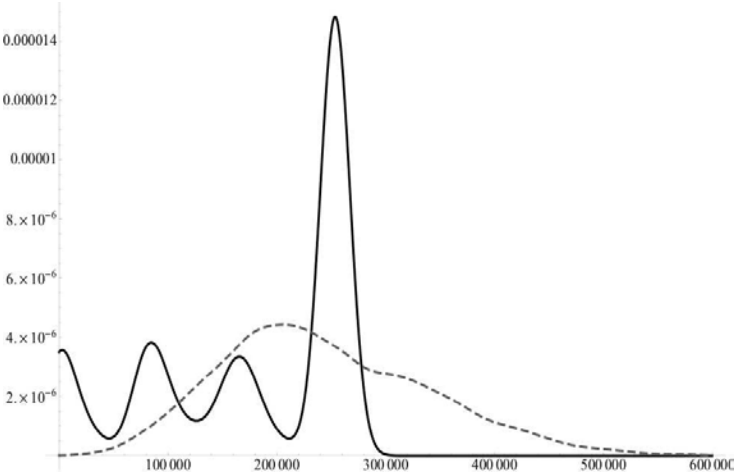
- a final payoff above $P^\flat = 1$ Meuros
- and a volume above $S^\flat = 40$ hm^3 in July and August

is about 90%

We plot iso-values for the maximal viability probability as a function of guaranteed thresholds $S^\flat$ and $P^\flat$



The probability distribution of the random gain reflects the viability objectives



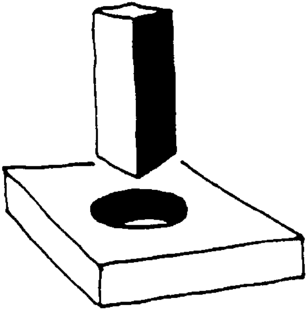
Outline of the presentation

In the resource managers literature, the distinction between objectives and decision rules is often blurred

In practice, we observe that resource managers generally

- design decision rules
- which directly incorporate objectives
- with confusion between objectives and decision rules

Mismatch can be avoided by highlighting the distinction between objectives and decision rules



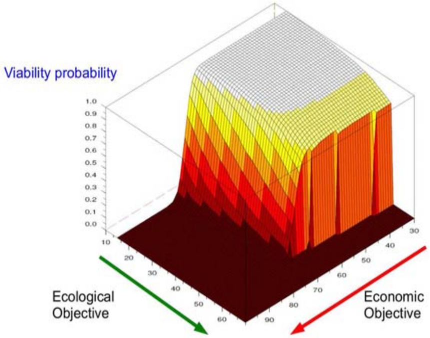
- Control theory makes a clear distinction between objectives and decision rules

$$\text{objectives} \Rightarrow \text{adapted decision rules}$$

- More specifically, viability theory puts emphasis on consistency between dynamics and objectives

$$\text{objectives} + \text{dynamics} \Rightarrow \text{decision rules}$$

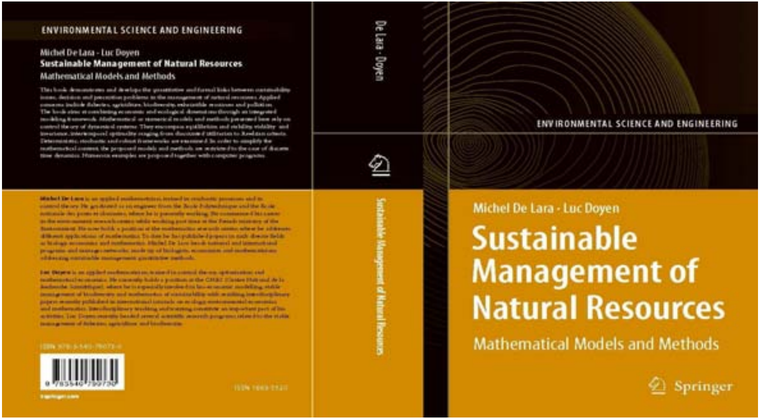
At the end of the day, where do we stand?



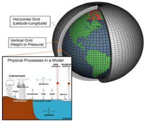
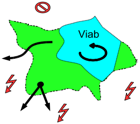
- Conceptual framework for quantitative sustainable management
- Managing ecological and economic conflicting objectives
- Ecosystem viable yields as a contribution to the "ecosystem approach"
- Displaying tradeoffs between ecology and economy sustainability thresholds and risk

“Nul n’est mieux servi que par soi-même”
“Self-promotion, nobody will do it for you” ;-)

M. De Lara, L. Doyen, Sustainable Management of Natural Resources. Mathematical Models and Methods, *Springer*, 2008.



THANK YOU!



Horizontal Grid (Latitude-Longitude)
Vertical Grid (Height or Pressure)
Physical Processes in a Model

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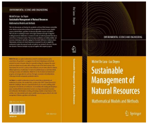
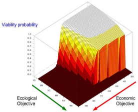
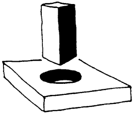
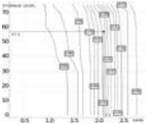
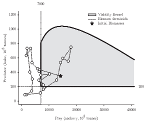
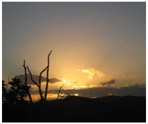
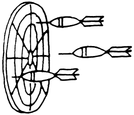
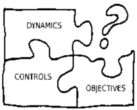

Michel De Lara · Luc Doyen
Sustainable Management of Natural Resources
Mathematical Models and Methods


Viability probability
Ecological Objective
Economic Objective


System


DYNAMICS
CONTROLS
OBJECTIVES